Nowhere-Zero 3-Flows and Odd $K_4$-Partitions

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Abstract. An odd $K_4$ is a subdivision of $K_4$ such that all four cycles corresponding to triangles in the $K_4$ are of odd length. It was shown by Catlin that every graph containing no odd $K_4$ is 3-colorable. The purpose of this paper is to establish the following dual version of Catlin’s theorem: every 2-edge-connected multigraph with no odd $K_4$-partition admits a nowhere-zero 3-flow, where an odd $K_4$-partition of a multigraph $G = (V, E)$ is a partition $\{V_1, V_2, V_3, V_4\}$ of $V$ such that (i) $G[V_i]$ is connected for each $1 \leq i \leq 4$; (ii) there is at least one edge between $V_i$ and $V_j$ for each pair $1 \leq i < j \leq 4$; and (iii) the number of edges between $V_i$ and $V \setminus V_i$ is odd for each $1 \leq i \leq 4$. (Co-authored with Xujin Chen and Wenan Zang)