



# Active Learning Through Classroom Applets in Linear Algebra and Geometry

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MAA Ohio Section, March 25, 2022










# GeoGebra

- Apps: <https://www.geogebra.org/>
  - Calculator Suite
  - 3D Calculator
  - CAS Calculator
  - Geometry
  - Graphing Calculator
  - Scientific Calculator
  - GeoGebra Classic
- Resources: <https://www.geogebra.org/materials>
- App downloads: <https://www.geogebra.org/download>



# Comparison of GeoGebra Apps

apps / features	 Scientific	 Graphing	 Geometry	 3D	 CAS	 Suite	 Classic
Numeric calculations	✓	✓	✓	✓	✓	✓	✓
Function operations	✓	✓	✓	✓	✓	✓	✓
Fraction operations	✓	✓	✓	✓	✓	✓	✓
Graphing		✓	✓	✓	✓	✓	✓
Sliders		✓	✓	✓	✓	✓	✓
Vectors & matrices		✓	✓	✓	✓	✓	✓
Table of values		✓			✓	✓	✓
Geometric constructions			✓	✓	✓	✓	✓
3D graphing				✓		✓	✓
Probability Calculator						✓*	✓
Derivatives & integrals				✓	✓	✓	✓
Equation solving				✓	✓	✓	✓
Symbolic calculations				✓	✓	✓	✓
Spreadsheet							✓

Source: <https://www.geogebra.org/m/shfwqcpr>



# GeoGebra subtleties

- GeoGebra Classic 5 v. GeoGebra Classic 6
  - Same features, but different graphical user interface (GUI).
  - GeoGebra Classic 6 looks like the online app
  - GeoGebra Classic 5 is preferred by authors who make use of dialog boxes and advanced commands
- GeoGebra file types
  - Activities
  - Books: Collections of activities
  - Lessons: A classroom version of a book or activity

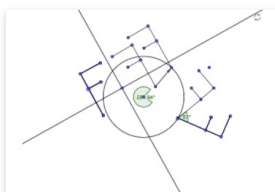


# Let's Try GeoGebra

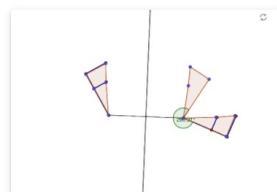
<https://www.geogebra.org/classroom/jzatmp4v>



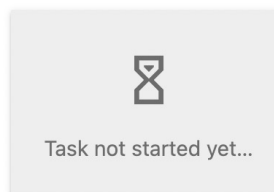
# Congruence via Transformations



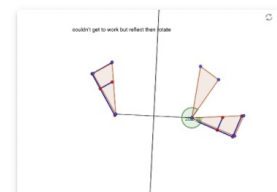
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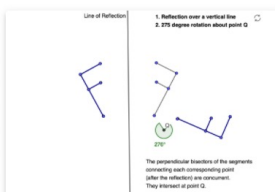
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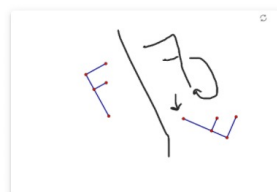
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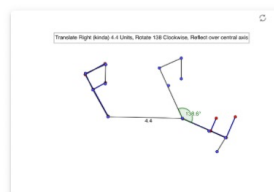
Student 4



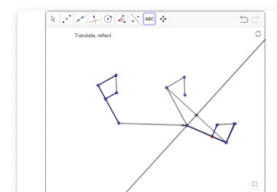
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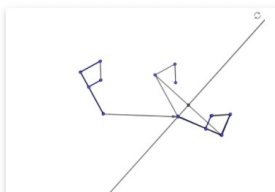
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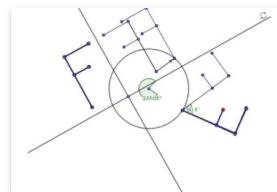
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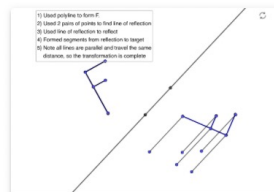
Student 8



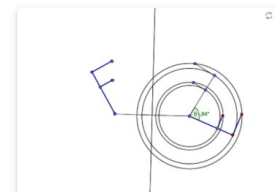
Student 9



Student 10



Student 11



Student 12



# Rectangle Diagonals

Determine condition(s) on the diagonals that will guarantee a rectangle.

Diagonals rods are joined below at point J.

3.03  
3.03

☒ Show Midpoints  
☒ Show Angle

**Student 1**

Diagonals rods are joined below at point J.

4.07  
4.07

☐ Show Midpoints  
☐ Show Angle

**Student 2**

Diagonals rods are joined below at point J.

3.57  
3.57

☐ Show Midpoints  
☐ Show Angle

**Student 3**

Diagonals rods are joined below at point J.

4.41  
4.41

☒ Show Midpoints  
☒ Show Angle

**Student 4**

Diagonals rods are joined below at point J.

5.07  
5.27

☒ Show Midpoints  
☐ Show Angle

**Student 5**

Diagonals rods are joined below at point J.

4.07  
4.07

☒ Show Midpoints  
☒ Show Angle

**Student 6**

Diagonals rods are joined below at point J.

3.05  
3.07

☒ Show Midpoints  
☒ Show Angle

**Student 7**

Diagonals rods are joined below at point J.

3.05  
3.05

☒ Show Midpoints  
☒ Show Angle

**Student 8**



# Rectangle Diagonals Reasoning

What condition(s) **on the diagonals** will guarantee a rectangle?

The intersection must still be at the midpoints of the diagonals but the diagonal lengths must be the same too.

Student 1

the midpoints of the diagonals have to intersect; the length of the diagonals have to have congruent lengths

Student 2

Both diagonals are the same length and we get a rectangle.

Student 3

Diagonal rods are congruent and that the diagonals intersect at the mid point J

Student 4

Same length diagonals with j being a midpoint

Student 5

If both diagonals are the same length, than this will guarantee a rectangle

Student 6

J is the midpoint and the lengths are equal

Student 7

Diagonals must intersect at the midpoints and opposite sides must be congruent

Student 8

Diagonals intersect at the midpoint and they are congruent

Student 9

The diagonals intersect at their midpoints.

Student 10

Both diagonals are the same length and have an intersection at both of the midpoints

Student 11

Point J is on the midpoint of both segments and the dotted sides equal 90 degrees. Parallel opposing sides

Student 12





# Desmos

- Teacher site: <https://teacher.desmos.com/>
- Student site: <https://student.desmos.com/>
- Graphing Calculator: <https://www.desmos.com/calculator>
  - Also available: scientific, four-function, and matrix calculators
- All of these available via <https://www.desmos.com/>
- Most featured activities are for middle school through Calculus.
  - <https://teacher.desmos.com/search?q=calculus>



# Let's Try Desmos

<https://student.desmos.com/join/8v8t7q>



# Solutions to a Differential Equation

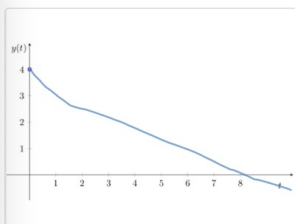
Screen 3 of 5

Draw a solution to the differential equation  $y'(t) = -k$ , given  $y(0) = 4$ .

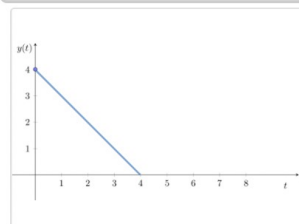
Responses

Overlay

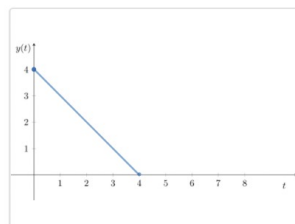
☐ Etta Zuber Falconer



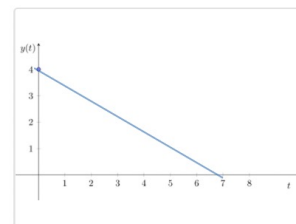
☒ Sarah Flannery



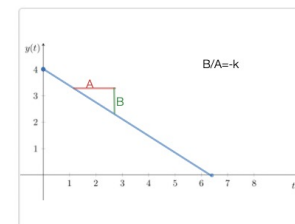
☐ Jean Springer



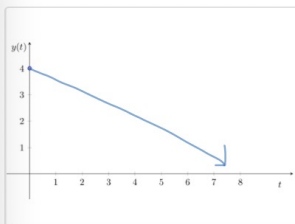
☐ John Urschel



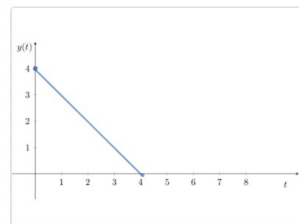
☐ Dorothy Vaughan



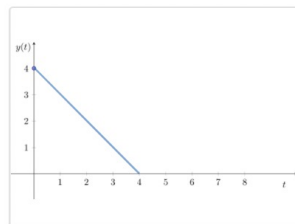
☐ Talitha Washington



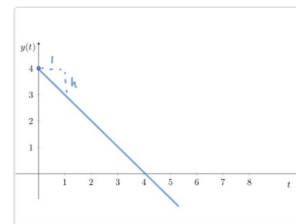
☐ Aryabhata



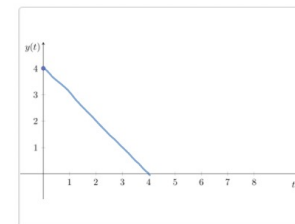
☐ Wang Zhenyi



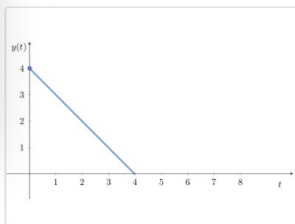
☐ Alexander Diaz-Lo...



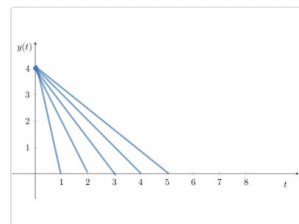
☐ Britney Gallivan



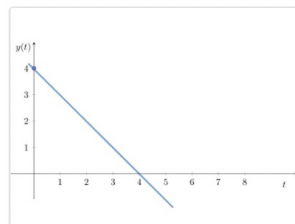
☐ Ibn al-Haytham



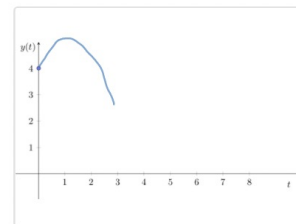
☐ Gloria Gilmer



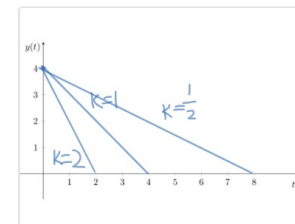
☐ Audrey Tang



☐ Karlie Noon



☐ Diarra Bousso Gueye

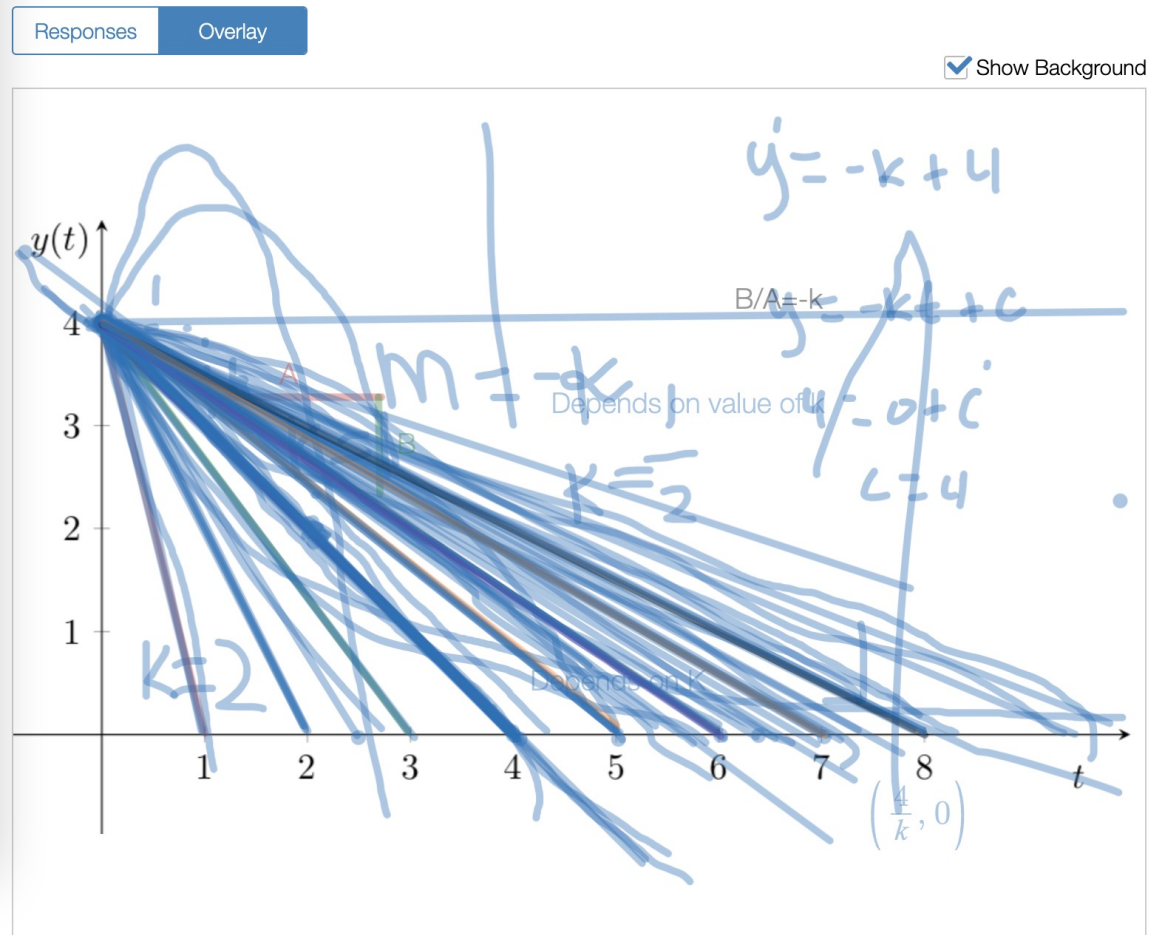




# Solutions to a Differential Equation, Overlay

Screen 3 of 5

Draw a solution to the differential equation  $y'(t) = -k$ , given  $y(0) = 4$ .





# Solutions to another Differential Equation

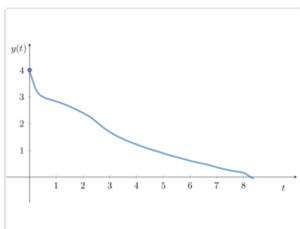
Screen 5 of 5

Draw a solution to the differential equation  $y'(t) = -(k/A)y(t)$ , given  $y(0) = 4$ .

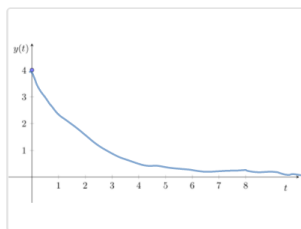
Responses

Overlay

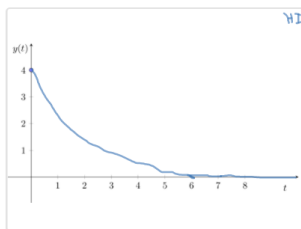
☐ Etta Zuber Falconer



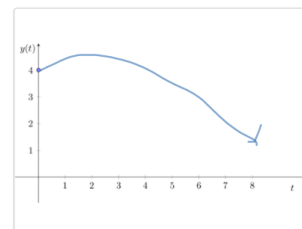
☐ John Urschel



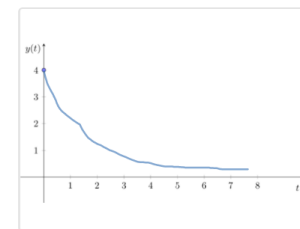
☐ Dorothy Vaughan



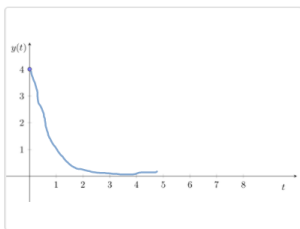
☐ Talitha Washington



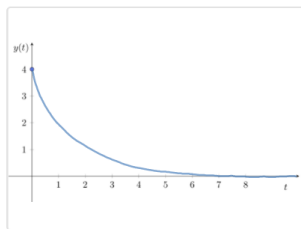
☐ Aryabhata



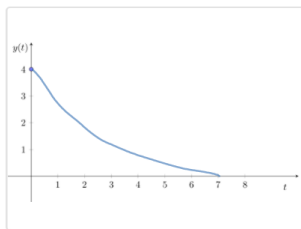
☐ Wang Zhenyi



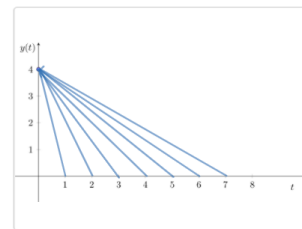
☐ Alexander Diaz-Lo...



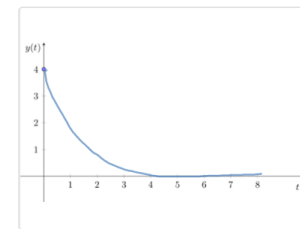
☐ Britney Gallivan



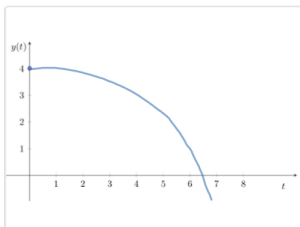
☐ Ibn al-Haytham



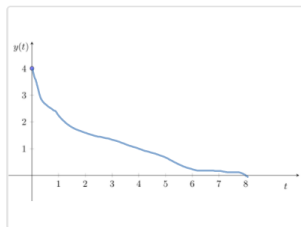
☐ Gloria Gilmer



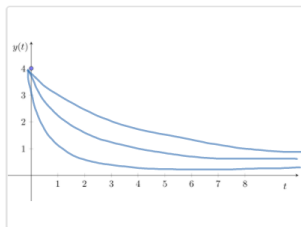
☐ Audrey Tang



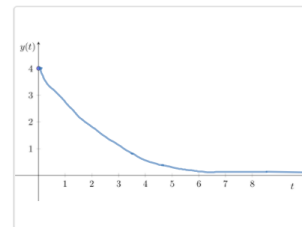
☐ Karlie Noon



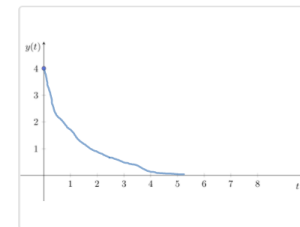
☐ Diarra Bousso Gueye



☐ Margaret H. Hamilton



☐ Gloria Conyers He...





# Geometry for Teachers

Write the Pythagorean Theorem.

☐ Grace Hopper

if the triangle is a right triangle, then  $a^2+b^2=c^2$

☐ Ibn al-Haytham

$a^2+b^2=c^2$

☐ Sophie Germain

$a^2+b^2=c^2$

☐ Hoang Xuan Sinh

$a^2+b^2=c^2$

☐ Kimberly Weems

$a^2+b^2=c^2$

☐ Cynthia Breazeal

$a^2+b^2=c^2$

Write a definition of rhombus. A rhombus is a quadrilateral ...

☐ Grace Hopper

with two pairs of parallel sides, equal length sides, and opposite angles equal

☐ Ibn al-Haytham

with opposite equal acute angles, opposite equal obtuse angles, and four equal sides.

☐ Sophie Germain

that has 2 pairs of parallel sides and equal side lengths.

☐ Hoang Xuan Sinh

and a parallelogram with opposite equal acute angles, and four sides that are equal in length.

☐ Kimberly Weems

a rhombus is a quadrilateral with four congruent sides



# Trigonometry

Find the sine, cosine, and tangent of an angle  $\theta$  with a terminal side through  $(-1, 2)$ .

1. Plot the point.
2. Sketch the angle.
3. Sketch the reference triangle.
4. Compute the sine, cosine, and tangent.

☐ Euphemia Lofton Haynes

$$\sin(\theta) = \frac{2}{\sqrt{5}}$$

$$\cos(\theta) = -\frac{1}{\sqrt{5}}$$

$$\tan(\theta) = \frac{2}{-1}$$

☐ Alan Turing

$$\sin \theta = \frac{2}{\sqrt{5}}$$

$$\cos \theta = -\frac{1}{\sqrt{5}}$$

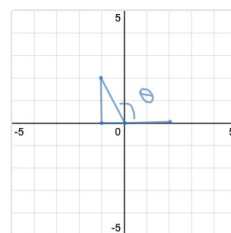
$$\tan \theta = -2$$

☐ Mae Jemison

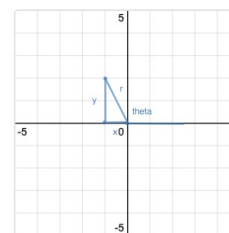
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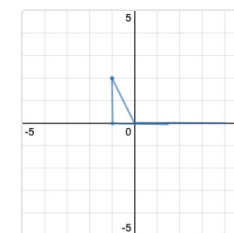
☐ Euphemia Loft...



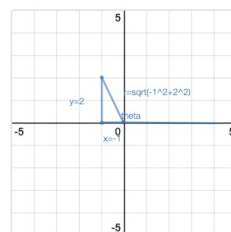
☐ Alan Turing



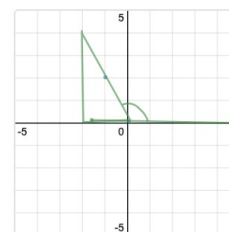
☐ Mae Jemison



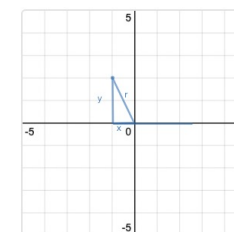
☐ Diana Ma



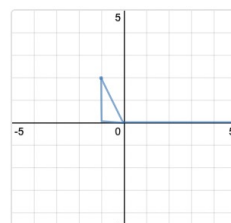
☐ Audrey Tang



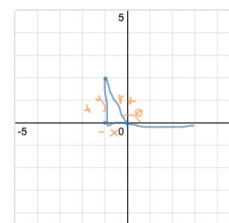
☐ Jagadish Chan...



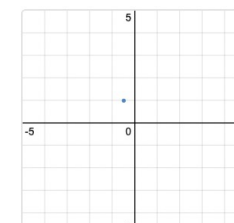
☐ Karlie Noon



☐ Vi Hart



☐ Gloria Gilmer





# Even, Odd, Neither, or Both

Is  $3/5$  Even, Odd, Neither, or Both?

Even

2 students  
Margaret H. Hamilton, Mary Somerville

Odd

2 students  
Wang Zhenyi, Ruth Gonzalez

Neither

22 students  
Sun-Yung Alice Chang, Wen-Tsun Wu, Julio Cesar de Mello e Souza, Audrey Tang, Britney Gallivan, Richard Tapia, Rochelle Gutierrez, Edray Goins, Emmy Noether, Al-Khwarizmi, Ada Lovelace, Erika Camacho, Hertha Ayrton, Diarra Bousso Gueye, Sophie Germain, Ahmes, Christine Mann Darden, Cynthia Breazeal, Arlie Petters, MC Escher, Gladys West, Pamela E. Harris

Both

0 students

☒ Margaret H. Hamilton

Even | Can be evenly divided by 2

☒ Mary Somerville

Even | Decimal is equal to .8 which is even

5 students selected

Present 4 Snapshots

Even |  
Decimal is equal to .8 which is even  
Mary Somerville

Even |  
Can be evenly divided by 2  
Margaret H. Ham...

Odd |  
because it is 0.3 and 3 is odd  
Wang Zhenyi

Odd |  
Because the 3 and 5 are both odd numbers  
Ruth Gonzalez

Snapshots below will not be presented

Neither |  
Can fractions be odd or even?

Send feedback to 5 students

☒

Send

Send and Close





# Even, Odd, Neither, or Both

Is 0 Even, Odd, Neither, or Both?

Even

12 students

Julio Cesar de Mello e Souza, Richard Tapia, Al-Khwarizmi, Erika Camacho, Diarra Bousso Gueye, Sophie Germain, Ahmes, Christine Mann Darden, Cynthia Breazeal, MC Escher, Gladys West, Pamela E. Harris

Odd

0 students

Neither

13 students

Sun-Yung Alice Chang, Wen-Tsun Wu, Margaret H. Hamilton, Britney Gallivan, Rochelle Gutierrez, Edray Goins, Emmy Noether, Mary Somerville, Ada Lovelace, Wang Zhenyi, Hertha Ayrton, Arlie Petters, Ruth Gonzalez

Both

1 student

Audrey Tang

☐ Julio Cesar de Mello e Souza

Even | I would consider any number that ends in 0 an even number

☐ Richard Tapia

Even | We just know 0 is an even number

6 students selected

Present 4 Snapshots

Neither | It's like asking if 1 is prime — it's just kind of a rule that it's not. If 0 is the ones place.  
Sun-Yung Alice ...

Neither | It is neither even or odd because 0 cannot be divided.  
Margaret H. Ham...

Neither | An even or odd number has to be a part of the counting numbers.  
Emmy Noether

Neither | It doesn't fit into either category  
Ruth Gonzalez

Snapshots below will not be presented

Neither | It's not odd or even because it has no factors

Both | No remainder when divided by two, but the result isn't

Send feedback to 6 students

✓

Send

Send and Close



# Resources

- My public GeoGebra activities available at
  - <https://www.geogebra.org/u/bfindell>
- MAA sample activities available at
  - <https://www.geogebra.org/m/qprp4rtq>
- Desmos MAA sample activities available at
  - <https://teacher.desmos.com/activitybuilder/custom/623dc90b257b040a795b401f>