MATH 2174: Autumn 2014
Midterm 1 Practice

1) Find all values $a$ so that the linear system has:
   i) no solutions;
   ii) one solution;
   iii) infinitely many solutions.

   \[
   \begin{align*}
   x + y &= 0 \\
   x - ay &= 3
   \end{align*}
   \]

2) Find all the values $x$ that make the matrix $A$ singular (without using the "determinant of the matrix $A$"):

\[
A = \begin{pmatrix}
2 & 4 & 1 \\
0 & 1 & 5 \\
2 & 4 & x
\end{pmatrix}
\]

3) Find the inverse $A^{-1}$ of the given matrix $A$ (without using the "determinant of the matrix $A$"):

\[
A = \begin{pmatrix}
1 & 1 \\
1 & 2
\end{pmatrix}
\]

4) Determine whether $W$ is a subspace of $\mathbb{R}^2$. Give a geometric description of $W$.

a) $W = \{x : x_1 + 4x_2 = 1\}$

b) $W = \{x : x_1x_2 = 1\}$
5) Let $\mathcal{N}(A)$ be the null space of the matrix $A$. Find a basis for $\mathcal{N}(A)$, and calculate the dimension of $\mathcal{N}(A)$.

$$A = \begin{pmatrix}
1 & 0 & 1 \\
1 & 1 & 2 \\
1 & 1 & 2 \\
\end{pmatrix}$$

6) Let $W = \text{Sp}\{S\}$ be the subspace of $\mathbb{R}^3$ spanned by the set $S$.

i) Find a subset $B$ of $S$ that is a basis for $W$.

ii) Express the vectors in $S$ that are not in $B$ as a linear combination of the vectors in $B$.

$$S = \left\{ \begin{pmatrix}
1 \\
1 \\
0 \\
\end{pmatrix}, \begin{pmatrix}
1 \\
2 \\
0 \\
\end{pmatrix}, \begin{pmatrix}
2 \\
3 \\
4 \\
\end{pmatrix}, \begin{pmatrix}
0 \\
0 \\
1 \\
\end{pmatrix}, \begin{pmatrix}
2 \\
1 \\
0 \\
\end{pmatrix} \right\}$$