1) Find all values $a$ so that the following linear system has:

i) no solutions;

ii) one solution;

iii) infinitely many solutions.

\[
\begin{align*}
  x + ay &= 0 \\
  x + y &= 3
\end{align*}
\]
2) Describe the set of all matrices that commute with the matrix

\[ A = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix} \]

I.e. find all the matrices \( B \) such that \( AB = BA \).
3) The general equation of a plane in $\mathbb{R}^3$ is

$$ax + by + cz + d = 0$$

Find a plane through the points $(1, 0, 1)$, $(1, 1, 0)$ and $(0, 3, 0)$. Is the plane unique?
4) Solve the equation

\[ Ax = 0 \]

where

\[
A = \begin{pmatrix}
3 & 0 & 1 \\
2 & 1 & 0 \\
10 & 2 & 2
\end{pmatrix}
\]

Give your answer in vector form.
5) True or false.
(a) If $A$ and $B$ are symmetric matrices, then $AB$ must also be symmetric.
(b) If $A$ and $B$ are nonsingular matrices, then $AB = BA$.
(c) If $v_1, v_2, ..., v_p$ are $m$-dimensional vectors, and if $p < m$, then the vectors
$\{v_1, v_2, ..., v_p\}$ are linearly independent.
(d) If $M$ and $N$ are square matrices such that $M$ is nonsingular and $MN = 0$.
then $N = 0$.
(e) If $x_1, x_2, ..., x_n$ are $m$-dimensional vectors that are all solutions of the
equation $Ax = 0$, then every linear combination $a_1x_1 + a_2x_2 + ... + a_nx_n$ is
also a solution of the same equation.
(f) If the matrix product $AB$ is well defined and square, then the matrix
product $BA$ is also well defined and square.
(g) A homogeneous system of 10 equations in 9 variables might have one
solution.
(h) If $v_1$ and $v_2$ are linearly dependent vectors in $\mathbb{R}^n$, and $A$ is an $(n \times n)$
matrix, then $Av_1$ and $Av_2$ are also linearly dependent.