1) Find all values $x$ such that the following matrix is defective:

$$A = \begin{pmatrix} x & 0 \\ 1 & 1 \end{pmatrix}$$

2) Find general (real-valued) solution for the following homogeneous linear system with constant coefficients. Plot a sample of trajectories in the phase plane.

$$x' = \begin{pmatrix} 4 & 6 \\ 3 & -4 \end{pmatrix} x$$

3) Solve the following initial value problem and plot a qualitative sample of the solution’s trajectory.

$$x' = Mx$$

$$x(0) = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

Where $M$ has eigenvalues $\lambda_1 = 1 + 2i$, $\lambda_2 = 1 - 2i$ and eigenvectors

$$\begin{pmatrix} 1 \\ i \end{pmatrix}, \begin{pmatrix} 1 \\ -i \end{pmatrix}$$

respectively.

4) In the following problem, the coefficient matrix contains a parameter $\alpha$. Determine the eigenvalues in terms of $\alpha$, and find the critical value(s) where
the qualitative nature of the system changes.

$$\begin{pmatrix} \alpha & 10 \\ -1 & -4 \end{pmatrix}$$