Midterm 1 **Math H5590 and 5111**

Having solved any 7 out of 9 problems would be enough. In your solutions you may use any fact proven in class, in the textbook, or in homework (excepting, of course, the fact you are assumed to prove now).

15% **1.** Draw the lattice of subgroups of the group $\mathbb{Z}_{36}$.

15% **2.** The abelian group $\mathbb{Z}_{12}^*$ has 4 elements. What group is it isomorphic to, $\mathbb{Z}_4$ or $V_4 = \mathbb{Z}_2^2$?

15% **3.** If two subgroups $H, K \leq G$ have relatively prime orders, prove that $H \cap K = 1$.

15% **4.** If G is a simple group and $\varphi: G \longrightarrow H$ is a homomorphism, prove that either φ is injective, or $\varphi(G) = 1$. (A group is said to be simple if it has no nontrivial proper normal subgroups.)

15% **5.** State 1st, 2nd, and 3rd isomorphism theorems.

15% **6.** Suppose that a finite group G acts on a set X, $x \in X$, and $\mathcal{O}_x = Gx$ is the orbit of x. Explain why $|\mathcal{O}_x| \mid |G|$. (You may use Lagrange's theorem.)

15% **7.** Let $H \leq G$, $|G : H| = n$, and assume that H contains no nontrivial subgroups that are normal in G. Prove that G is isomorphic to a subgroup of S_n.

15% **8.** Let $H \leq G$ be a subgroup of finite index and assume that the number of subgroups conjugate to H in G (including H itself) is equal to $|G : H|$. Prove that $N_G(H) = H$.

15% **9.** Determine how many elements of S_5 commute with the permutation $(1, 2)(3, 4)$.