Midterm 2 **Math H5590 and 5111**

In your solutions you may use any fact proven in class, in the textbook, or in homework (except for the very fact you are assumed to prove here).

20% **1.** (a) Find all (up to isomorphism) abelian groups of order $600 = 2^3 \cdot 3 \cdot 5^2$.

(b) Determine which of these groups the group $\mathbb{Z}_{30} \times \mathbb{Z}_{20}$ is isomorphic to.

15% **2.** Prove that for any prime p there exists a nonabelian group of order p^3. (Don't construct it explicitly, just prove that it exists.)

20% **3.** The matrix $\left(\begin{smallmatrix} 0 & 10 \\ 1 & 3 \end{smallmatrix}\right)$ with entries from $\mathbb{Z}_{11}$ has order 5 in $\mathrm{GL}_2(\mathbb{F}_{11})$. Construct, in terms of generators and relations, a nonabelian group of order $605 = 11^2 \cdot 5$.

15% **4.** State Sylow's theorem(s), as many parts as you can.

15% **5.** Prove that all groups of order $1225 = 5^2 \cdot 7^2$ are abelian.

20% **6.** Let G be a nonabelian group of order 55.

(a) How many elements of order 5 does G have?

(b) If a and b are two elements of order 5 in G, prove that b is conjugate to a^k for some k.

15% **7.** Prove that all groups of order $19965 = 3 \cdot 5 \cdot 11^3$ are solvable.