TOPICS IN REPRESENTATION THEORY

Course Information

Instructor. Sachin Gautam. Office MW 640. gautam.42@osu.edu

Office hours. Mondays, Fridays 10-11AM, or by appointment.

Class time and place. MWF 1.50-2.45PM. University Hall 043.

Homepage. https://people.math.osu.edu/gautam.42/A19/topics.html

Lecture notes. This course will be based entirely on lectures. I will upload my notes at the following link. So, in case you miss a class (**not recommended**) you can download the notes.

https://people.math.osu.edu/gautam.42/A19/notes.html

Homework. See the course schedule on page 3, for the tentative dates of posting and submission deadlines of the homework sets. These will be available at:

https://people.math.osu.edu/gautam.42/A19/homework.html

Individual projects. I will make a list of topics, related to the core of this course, but which will not be covered (due to finiteness of time). You are welcome to choose and learn about one of your liking. Given the size of the class, not every *individual project* will get a chance of in–class presentation (I will try to accommodate as many as possible, but some may have to be content with submitting a write–up of the topic).

https://people.math.osu.edu/gautam.42/A19/talks.html

Prerequisites. I will keep the prerequisites for this course to a minimum: basic complex analysis (up to Cauchy's residue theorem), and linear algebra (tensor product of vector spaces, matrices: Jordan canonical form). Some familiarity with the language of category theory will be beneficial, but not strictly necessary.

Overview

The aim of this course is to understand the interplay between certain consistent systems of PDE's and algebraic objects, known as *quantum groups.* These differential equations originated from mathematical physics (conformal field theory, statistical mechanics) and the corresponding algebraic structures describe their monodromy (roughly speaking).

With the additional goal of keeping the lectures more-or-less self-contained, this course can be (more modestly speaking) viewed as a gentle introduction to the following list of topics (see page 3 for a tentative course schedule).

(1) ODE's in the complex plane.
(2) Hyperplane arrangements and rational connections.
(3) Root systems. Braid groups.
(4) De Concini–Procesi associators.
(5) Lie algebras and representations.
(6) Knizhnik–Zamolodchikov and Casimir connections.
(7) Braided tensor categories.
(8) Quantum groups and representations.
(9) Deformation theory.

General Policies

Academic Misconduct. It is the responsibility of the Committee on Academic Misconduct to investigate or establish procedures for the investigation of all reported cases of student academic misconduct. The term "academic misconduct includes all forms of student academic misconduct wherever committed; illustrated by, but not limited to, cases of plagiarism and dishonest practices in connection with examinations. Instructors shall report all instances of alleged academic misconduct to the committee (Faculty Rule 3335-5-487). For additional information, see the Code of Student Conduct (http://studentaffairs.osu.edu/info_for_students/csc.asp).

Disability Services. Students with disabilities that have been certified by the Office for Disability Services will be appropriately accommodated and should inform the instructor as soon as possible of their needs. The Office for Disability Services is located in 150 Pomerene Hall, 1760 Neil Avenue; telephone 292-3307, TDD 292-0901; http://www.ods.ohio-state.edu/

Course Schedule

The following schedule is tentative only. You will be notified of any changes by email, or in class. The most recent version of this syllabus will remain available at
https://people.math.osu.edu/gautam.42/A19/syllabus.pdf

Week 1. (August 21,23) Introduction. ODE's over $\mathbb{C}$. Regular and regular singular points. Monodromy. Drinfeld associator.

Week 2. (August 26-30) No class.

Week 3. (Sept. 4,6 *Holiday: September 2 Monday Labour day*) Asymptotic expansions. Irregular singularities. Stokes' phenomenon.
Homework 1. Due on Sept. 20 Friday.

Week 4. (Sept. 9-13) Several variables. Kohno's lemma. Hyperplane arrangements. Configuration space and Artin's braid group.

Week 5. (Sept. 16-20) Root systems. Definition and classification theorem. Weyl group. Braid groups and their affine analogues.
Homework 2. Due on Oct. 4 Friday.

Week 6. (Sept. 23-27) De Concini–Procesi associators. Lie algebras and their representations.

Week 7. (Sept. 30 - Oct. 4) Lie algebras and their representations continued. Weyl character formula.
Homework 3. Due on Oct. 18 Friday.

Week 8. (Oct. 7,9 *Holiday: Oct. 10-13 Autumn break*) KZ and Casimir equations. Fusion operator via differential equations.

Week 9. (Oct. 14-18) Braided tensor categories. Coxeter categories.

Week 10. (Oct. 21-25) Language of categories continued. Quantum groups: first example. Hopf algebras and R-matrices.
Homework 4. Due on Nov. 8 Friday.

Week 11. (Oct. 28 - Nov. 1) Quantum groups continued: R-matrix and quantum Weyl group.

Week 12. (Nov. 4-8) Quantum groups continued. Verification of the axioms of Coxeter categories.

Week 13. (Nov. 13,15 *Holiday: Nov. 11 Veteran's day*) Deformations of algebras, Hochschild cohomology.

Week 14. (Nov. 18-22) Deformation theory continued. Lie algebra cohomology, Whitehead's lemmas.
Homework 5. Due on Dec. 6 Friday.

Week 15. (Nov. 25 *Holiday: Thanksgiving break.*) Kohno–Drinfeld theorem.

Week 16. (Dec. 2,4 *Last day of classes: Dec. 4th.*) Discussion.