APPLIED COMPLEX VARIABLES I (MATH 7651)

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Course Information

Homepage. https://people.math.osu.edu/gautam.42/Au23/complex.html

Class time and place. 9.10am - 10.05am, Caldwell Lab 115.

Office hours. Mondays 11am - 1pm, MW 640.

Homework and grading. Your grade will be determined by your participation in the class, during office hours, and a final presentation. I will compile a list of problems:
https://people.math.osu.edu/gautam.42/Au23/Problems.pdf

Class participations means that you should attempt and discuss (some of) these problems with me, during office hours, and your classmates.

Textbook. The course will be based on lecture notes, which will be regularly uploaded at the following link: https://people.math.osu.edu/gautam.42/Au23/notes.html

For reference, I will be using the following texts.

- M.J. Ablowitz and A.S. Fokas, *Complex variables: introduction and applications.*
- O. Costin, *Asymptotics and Borel summability.*
- R. Courant, *Dirichlet's principle, conformal mapping and minimal surfaces.*
- Z. Nehari, *Conformal mappings.*
- T. Ransford, *Potential theory in the complex plane.*
- L.I. Volkovyskii, G.L. Lunts and I.G. Aramanovich, *A collection of problems on complex analysis.*
- E.T. Whittaker and G.N. Watson, *A course of modern analysis.*

Contents

This course is the first of a year–long sequence of two courses. It is aimed at giving a thorough treatment of various applications of complex analysis, to conformal geometry, asymptotic analysis, special functions and systems of difference, differential and integral equations. [1]

Part 1. *Review of foundational results.*

- Functions of a complex variable. Complex differentiability. Cauchy–Riemann equations. Cauchy's theorem and integral formula. Liouville's theorem and fundamental theorem of algebra. The argument principle and winding number. Rouché's theorem.
- Taylor and Laurent series expansions. Notion of uniform convergence and Weierstrass' theorem. Power series, Abel's theorem (radius of convergence). Open mapping and inverse function theorems. Poles and essential singularities.
- Weiestrass' notion of analytic continuation and Riemann surfaces. Schwarz' reflection principle. Infinite sum/product expansions à la Mittag–Leffler and Weierstrass.

Part 2. *Difference–differential equations.*

- Functions defined via integrals. Laplace transform and its basic properties.

[1] Every problem in mathematics eventually reduces to something in combinatorics, linear algebra or complex analysis. –Kapil Hari Paranjape.

- Divergent series. Asymptotic series à la Poincaré. Methods of computing asymptotic expansions: Watson's lemma, steepest descent method, WKB method. Perturbative techniques.
- Difference equations. Euler's gamma function. Stirling series. Monodromy of difference equations and Stokes' phenomenon.
- Differential equations. Frobenius' method. Monodromy computations. Gauss' hypergeometric function. Irregular singularities and Stokes' multipliers.

Part 3. *Conformal geometry.*

- Conformal equivalence. Möbius transformations. Schwarz' lemma. Geometric and algebraic properties of Möbius transformations.
- Digression: Arzelà–Ascoli and Montel's theorems. Families of holomorphic functions. Hurwitz' theorem.
- Riemann mapping theorem. Koebe's proof using Montel and Hurwitz' theorems.
- Dirichlet boundary value problem. The case of the unit disc - Poisson kernel. Hilbert's solution to Dirichlet's problem. Weyl's regularity lemma.
- Potential theory. Harmonic and subharmonic function à la Perron. Optimization problems. Notion of capacity and related measures. Polar sets.
- Schwarz–Christoffel formula. Mapping properties of hypergeometric function and elliptic functions.

Preview of coming attractions: for Spring 2024

Part 4. *Elliptic functions.* Basic properties of doubly–periodic functions. Weierstrass' $\wp$–function. Jacobi's θ–function. Elliptic integrals. $\mathrm{SL}_2(\mathbb{Z})$–action on $\mathbb{H}$. Modular forms.

Part 5. *Riemann surfaces.* Uniformization theorem. $\mathbb{P}^1 \setminus \{0, 1, \infty\}$ and Fuchsian groups. Picard's big theorem.

Part 6. *Non–linear equations.* Singular integral equations. Riemann–Hilbert factorization problem. Plemelj formula. Inverse monodromy problem. Movable singularities and Painlevé transcendents.

General Policies

Academic Misconduct. It is the responsibility of the Committee on Academic Misconduct to investigate or establish procedures for the investigation of all reported cases of student academic misconduct. The term "academic misconduct includes all forms of student academic misconduct wherever committed; illustrated by, but not limited to, cases of plagiarism and dishonest practices in connection with examinations. Instructors shall report all instances of alleged academic misconduct to the committee (Faculty Rule 3335-5-487). For additional information, see the Code of Student Conduct (http://studentaffairs.osu.edu/info_for_students/csc.asp).

Disability Services. Students with disabilities that have been certified by the Office for Disability Services will be appropriately accommodated and should inform the instructor as soon as possible of their needs. The Office for Disability Services is located in 150 Pomerene Hall, 1760 Neil Avenue; telephone 292-3307, TDD 292-0901; http://www.ods.ohio-state.edu/