LIE ALGEBRAS (MATH 7161)

Instructor. Sachin Gautam. Office MW 640. gautam.42@osu.edu

Course Information

Homepage. https://people.math.osu.edu/gautam.42/Au26/lie.html

Class time and place. 12.40-1.35pm, MWF: Enarson 218.

Office hours. MW: 10.30am-12noon.

Textbook. The course will be based entirely on intructor's lecture notes.
https://people.math.osu.edu/gautam.42/Au26/notes.html

I will use Prof. Etingof's notes for reference, which can be found at the following link.
https://arxiv.org/abs/2201.09397

Grading. Your final grade will be based on homework assigments. There will be around $7 \sim 8$ homework assignments throughout the semester (roughly one every two weeks). Your solutions should be clear, legible and handwritten.

General Policies

Academic Misconduct. It is the responsibility of the Committee on Academic Misconduct to investigate or establish procedures for the investigation of all reported cases of student academic misconduct. The term "academic misconduct includes all forms of student academic misconduct wherever committed; illustrated by, but not limited to, cases of plagiarism and dishonest practices in connection with examinations. Instructors shall report all instances of alleged academic misconduct to the committee (Faculty Rule 3335-5-487). For additional information, see the Code of Student Conduct (http://studentaffairs.osu.edu/info_for_students/csc.asp).

Disability Services. Students with disabilities that have been certified by the Office for Disability Services will be appropriately accommodated and should inform the instructor as soon as possible of their needs. The Office for Disability Services is located in 150 Pomerene Hall, 1760 Neil Avenue; telephone 292-3307, TDD 292-0901; http://www.ods.ohio-state.edu/

Course Schedule

The following schedule is tentative only (especially Week 13 onwards). You will be notified of any changes by email, or in class. The most recent version of this syllabus will remain available at
https://people.math.osu.edu/gautam.42/Au26/syllabus-lie.pdf

Week 1. (August 26,28). Definition of a Lie algebra, subalgebra, ideal. Homomorphisms of Lie algebras. Representations. Adjoint representation. Examples.

Week 2. (August 31, September 2,4). Enveloping algebra of a Lie algebra. Some general remarks on Hopf algebras and monoidal categories.

Week 3. (Sept. 7 is a holiday. Sept 9, 11). PBW theorem and free Lie algebra. Solvable and nilpotent Lie algerbas.

Week 4. (Sept. 14,16,18). Theorems of Engel and Lie. Invariant forms. Review of some linear algebra.

Week 5. (Sept. 21,23,25). Jordan-Chevalley decomposition. Invariant forms and reductive Lie algebras. Cartan's criterion.

Week 6. (Sept. 28, 30; Oct. 2) Extensions and Lie algebra cohomology. Weyl's complete reducibility theorem.

Week 7. (Oct. 5,7,9) Representation theory of sl(2).

Week 8. (Oct. 12,14. Oct. 16 is Autumn break). Cartan subalgebras and root systems. Weyl group.

Week 9. (Oct. 19,21,23). Classification of root systems. Cartan–Killing structure theorem.

Week 10. (Oct. 26,28,30). Uniqueness of Cartan subalgebras. Examples of simple finite-dimensional Lie algebras.

Week 11. (Nov. 2,4,6). Representation theory of finite-dimensional, simple Lie algebras. Verma modules and Shapovalov form.

Week 12. (Nov. 9,13. Nov. 11 is Veteran's day). Weyl group actions and classification of finite-dimensional, irreducible representations.

Week 13. (Nov. 16,18,20). Lie bialgebras and quantization. Quantum groups associated to finite-dimensional, simple Lie algebras. Deformation theory.

Week 14. (Nov. 23. Nov. 25,27 - Thanksgiving break). Whitehead's lemma.

Week 15. (Nov. 30, Dec. 2,4). Drinfeld's double method. Hopf pairings and R-matrices.

Week 16. (Dec. 7,9. Dec. 9 is the last day of classes). Discussion of further topics in Lie theory.