ALGEBRA 1: PROBLEM SET 8

Problem 1. Compute the character table of S_5.

Problem 2. Let W be the irreducible 2–dimensional representation of S_4 (see Lecture 19 page 4). Compute the decomposition of $\mathrm{Ind}_{S_4}^{S_5}(W)$ into direct sum of irreducible representations of S_5. Here we view S_4 as a subgroup of S_5 consisting of permutations which fix the element 5.

Problem 3. Prove that every finite–dimensional irreducible representation of the dihedral group D_n is either 1 or 2–dimensional.

Problem 4. Let V be the (unique) irreducible representation of S_5 of dimension 6 (from problem 1). Consider $S_3 \times S_2$ as a subgroup of S_5 consisting of permutations which preserve the decomposition $\{1, \ldots, 5\} = \{1, 2, 3\} \sqcup \{4, 5\}$. Compute the decomposition of $\mathrm{Res}_{S_3 \times S_2}^{S_5}(V)$ into direct sum of irreducible representations of $S_3 \times S_2$.

Problem 5. Let $S_{n-1} \subset S_n$ be the subgroup consisting of permutations fixing n. Let $V \subset \mathbb{C}^n$ be the $n-1$–dimensional irreducible representation of S_n (see Lecture 20 section (20.1)). How many irreducible summands does $\mathrm{Res}_{S_{n-1}}^{S_n}(V)$ have?