ABSTRACT ALGEBRA I - MATH 6111

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Course Information

Homepage. https://people.math.osu.edu/gautam.42/F17/algebra.html

Class time and place. Mondays, Wednesdays, Fridays 11.30AM-12.25PM. Smith Lab 2180

Recitations. Tuesdays, Thursdays 11.30AM-12.25PM. University Hall 043

Course description. This is the first semester of the PhD level abstract algebra sequence. It is now part of the qualifying exam system. To receive credit for the course as part of the qualifying exam structure, you must obtain an A- or better.

Contents. The following topics are considered as the syllabus for the qualifying exam.

- *Group Theory.* Monoids, subgroups, cyclic groups, homomorphisms, normal subgroups, factor groups, direct products, group actions, counting lemmas, Sylow theorems, composition series, Jordan-Hölder theorem, solvable and nilpotent groups, semi-direct products, permutation group.

- *Representations.* Complex linear representations of finite groups, complete reducibility, characters, orthogonality of characters, regular representation, permutation representation, induction and restriction, Frobenius reciprocity.

- *Ring Theory.* basic definitions of rings, ideals and modules, commutative rings, prime ideals, Noetherian and Artinian rings, Hilbert basis theorem, localization, integral domains, PID, UFD, irreducibility criteria, symmetric polynomials, discriminant.

Grading. The grade will be based on homework assignments (25%), one take-home exam (10%), one mid term (25%) and the final (40%).

- *Homework.* I will assign and collect homework weekly. Typically that will be a Friday, with few exceptions (see Course Schedule on page 3). You are encouraged to work together, but your write ups must be your own. **Late homework will not be graded**.

- *Exams.* There will be three exams: a take home, a midterm and the final. Dates of these exams are included in the Course Schedule (page 3). The last two will be closed-book, in-hall exams - similar to the algebra qualifying.

- *Class Participation and Attendance.* Although attendance is not regularly monitored frequent absences are likely to be noted and may factor into the grade in borderline cases.

Prerequisites. We will keep the prerequisites to a minimum, while Master's level Algebra II (5112) is the official requirement for this course. The representations section of the course will assume familiarity with some linear algebra, but I will review any material that is needed.

Suggested texts

The course will be based entirely on the lecture notes. If you miss a class (not recommended) you can download the notes at the following

https://people.math.osu.edu/gautam.42/F17/notes.html

There are a lot of excellent textbooks on abstract algebra. The following list of recommended texts is to provide some (optional) reading suggestions, and more examples/problems to work out.

- S. Lang, *Algebra*, 3rd edition, Springer.
- N. Bourbaki, *Algebra Chapters 1-3*, Springer.
- N. Jacobson, *Basic Algebra I, II*, Dover.
- J-P. Serre, *Linear representations of finite groups*, Springer.
- M. Atiyah and I. Macdonald *Introduction to commutative algebra*, Addison Wesley.

General Policies

Academic Misconduct. It is the responsibility of the Committee on Academic Misconduct to investigate or establish procedures for the investigation of all reported cases of student academic misconduct. The term "academic misconduct includes all forms of student academic misconduct wherever committed; illustrated by, but not limited to, cases of plagiarism and dishonest practices in connection with examinations. Instructors shall report all instances of alleged academic misconduct to the committee (Faculty Rule 3335-5-487). For additional information, see the Code of Student Conduct (http://studentaffairs.osu.edu/info_for_students/csc.asp).

Disability Services. Students with disabilities that have been certified by the Office for Disability Services will be appropriately accommodated and should inform the instructor as soon as possible of their needs. The Office for Disability Services is located in 150 Pomerene Hall, 1760 Neil Avenue; telephone 292-3307, TDD 292-0901; http://www.ods.ohio-state.edu/

Course Schedule

The following schedule is tentative only. You will be notified of any changes by email, or in class. The most recent version of this syllabus will remain available at
https://people.math.osu.edu/gautam.42/F17/syllabus.pdf

Week	Topics	Homework deadline
1 8/22-25	Definitions (Groups, homomorphisms, normal subgroups). Presentation of a group. Groups actions. Counting lemmas.	HW 1 due on 9/1
2 8/28-9/1	Quotient groups. Simple groups. Basic Isomorphism Theorems	HW 2 due on 9/8
September 4 - Labour day, no class		
3 9/4-9/8	Short exact sequences. Semidirect products. Sylow Theorems.	HW 3 due on 9/15
4 9/11-15	Composition series. Jordan-Hölder theorem Derived groups. Solvable and nilpotent groups.	HW 4 due on 9/22
5 9/18-22	Symmetric groups. Simplicity of alternating groups.	HW 5 due on 9/27 **Wednesday**
First exam (take home) will be assigned on 9/28, due 9/29		
6 9/25-29	Basic definitions of representation theory. $\oplus, \otimes$ and duals. Irreducible representations. Complete reducibility theorem.	HW 6 due on 10/6
7 10/2-6	Regular representation. characters and orthogonality. Decomposition of regular representation.	HW 7 due on 10/11 **Wednesday**
8 10/9-13	Character table. Induced representations.	HW 8 due on 10/20
October 12,13 - Fall break, no classes		
9 10/16-20	Frobenius reciprocity. Mackey's theorem A glimpse of the representation theory of S_n.	HW 9 due on 10/27
10 10/23-27	Definitions (rings, ideals, modules). Basic isomorphism results. Finite generation. Integral domains. Characteristic of a ring.	No homework
In class midterm exam on 10/30 Monday.		
11 10/30-11/3	Commutative rings. Prime and maximal ideals. CRT.	HW 10 due on 11/09 **Thursday**
12 11/6-10	Polynomial rings. Algebraic dependence. Several variables. Modules - direct sums, tensor products, exact sequences.	HW 11 due on 11/17
November 10 - Veteran's day, no class		
13 11/13-17	Localization. Noetherian and Artinian rings. Hilbert Basis Theorem.	HW 12 due on 11/27 **Monday**
14 11/20-24	Artinian local rings.	No homework
November 22-24 - Thanksgiving break, no classes		
15 11/27-12/1	Principal ideal domains. UFD. Dedekind domains.	HW 13 due on 12/6 **Wednesday**
16 12/4-6	A glimpse of algebraic number theory.	

Final exam: Thursday 12/14. 10-11.45AM. Location TBD.