PRACTICE MID TERM II: COMPLEX VARIABLES

(1) Determine whether the series $\sum_{n=2}^{\infty} \sin\left(\frac{\pi}{n}\right)$ converges or not.

(2) Find the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{n!}{n^n} z^n$.

(3) Prove that the following identity holds for $|z| < 1$:

$$\sum_{n=1}^{\infty} n^2 z^n = \frac{z(1+z)}{(1-z)^3}$$

(4) Compute the following residue $\mathrm{Res}_{z=0}\left(\frac{1}{z^4(1-z)(2-z)}\right)$.

(5) Compute the integral $\int_0^{2\pi} \frac{1}{2+\cos(\theta)}\, d\theta$.

(6) Compute the following infinite integral:

$$\lim_{R\to\infty} \int_{-R}^{R} \frac{x}{(x^2+4x+13)^2}\, dx$$