LIE GROUPS AND THEIR REPRESENTATIONS

SACHIN GAUTAM

Course Information

Class time. Mondays, Wednesdays, Fridays 12.40-1.35PM

Location. Journalism Bldg. (JR) 221.

Office hours. Mondays, Wednesdays, Fridays 2-3PM (Office MW 640). Or, by appointment: email `gautam.42@osu.edu`

Course description and goals. This course provides students with a solid working knowledge in the theory of Lie groups and their representations. We will primarily focus on analytic and algebraic aspects of compact Lie groups through a wide variety of examples. These objects themselves and techniques used to study them are prevalent in several areas of mathematics. This course should be of interest to students working in or planning to work in representation theory, number theory or algebraic geometry.

Grading. The grade will be based on homework assignments and final presentations.

- There will be approximately 10 homework assignment sheets. Due dates of assignments will be announced and set typically a week after the assignments are published.

- Besides homework assignments, the grade will be based on a final in-class short presentation. You will be asked to write up and submit notes for your lecture. A list of suggested topics will be published about three to four weeks before the end of classes and student presentations will be organized during the finals week.

- Class Participation and Attendance: Although attendance is not regularly monitored frequent absences are likely to be noted and may factor into the grade in borderline cases.

Prerequisites. Differentiable manifolds and Lie algebras are the official prerequisites for this course. However, I will try not to assume anything other than basic topology, linear algebra and some complex analysis. In any case, I will provide a review for each topic as we need it.

Course schedule

The following schedule is tentative only and subject to change.

Week 1. Introduction to Lie groups and Lie algebras. Review of some differential geometry: manifolds, vector fields, differential forms.

Week 2. Tangent bundle of a Lie group. Adjoint action. Exponential map. Correspondence between Lie groups and Lie algebras - Lie's theorems.

Week 3. Some general results on Lie algebras: Jordan decomposition, solvable and nilpotent Lie algebras.

Week 4. Semisimple Lie algebras. Cartan-Killing form. Cartan subalgebras. Root systems.

Week 5. Classification of root systems. Chevalley involution and compact real forms.

Week 6. Introduction to representation theory. Correspondence between representations of Lie groups and Lie algebras. Compact Lie groups.

Week 7. Several examples of Lie groups.

Week 8. Compact Lie groups continued. Maurer-Cartan differential form. Integration on compact Lie groups - Weyl's unitary trick.

Week 9. Complete reducibility theorem. Orthogonality of irreducible characters. Weyl's integration formula.

Week 10. Function spaces associated to compact Lie groups. Review of some functional analysis. Peter-Weyl theorem.

Week 11. Classification of irreducible representations. Weyl's character formula.

Week 12-13. Group actions on manifolds. Homogeneous spaces. Explicit construction of irreducible representations - Borel-Weil-Bott theorem.

Week 14. Decomposition of tensor product into irreducible constituents. Towards combinatorial identities involving tensor product decompositions.

Suggested reading

The course will be based entirely on the instructor's lectures. The lecture notes will be made available on the instructor's homepage. There are a lot of excellent textbooks on Lie groups and representation theory. The following list of recommended texts is to provide some optional reading suggestions:

- C. Chevalley, *Theory of Lie groups.* Princeton Landmarks in Mathematics. Princeton University Press (1946).

- A. W. Knapp, *Lie groups: beyond an introduction.* Progress in Mathematics, volume 140. Birkhäuser Boston (1996).

- J. Milnor, *Remarks on infinite–dimensional Lie groups.* in Relativity, groups and topology II, Les Hauches session XL (1983).

- T. Bröcker and T. tom Dieck *Representations of compact Lie groups.* Graduate Texts in Mathematics 98. Springer-Verlag (1985).

General Policies

Academic Misconduct. It is the responsibility of the Committee on Academic Misconduct to investigate or establish procedures for the investigation of all reported cases of student academic misconduct. The term "academic misconduct includes all forms of student academic misconduct wherever committed; illustrated by, but not limited to, cases of plagiarism and dishonest practices in connection with examinations. Instructors shall report all instances of alleged academic misconduct to the committee (Faculty Rule 3335-5-487). For additional information, see the Code of Student Conduct (http://studentaffairs.osu.edu/info_for_students/csc.asp).

Disability Services. Students with disabilities that have been certified by the Office for Disability Services will be appropriately accommodated and should inform the instructor as soon as possible of their needs. The Office for Disability Services is located in 150 Pomerene Hall, 1760 Neil Avenue; telephone 292-3307, TDD 292-0901; http://www.ods.ohio-state.edu/