ABSTRACT ALGEBRA II - MATH 6112

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Course Information

Homepage. https://people.math.osu.edu/gautam.42/S18/algebra.html

Class time and place. Mondays, Wednesdays, Fridays 11.30AM-12.25PM. Enarson EC 322.

Recitations. Tuesdays, Thursdays 11.30AM-12.25PM. Enarson EC 322.

Course description. This is the second semester of the PhD level abstract algebra sequence. It is now part of the qualifying exam system. To receive credit for the course as part of the qualifying exam structure, you must obtain an A- or better.

Contents. The following topics are considered as the syllabus for the qualifying exam.

- *Category theory.* Categories, functors, natural transformations. Equivalence of categories. Yoneda's lemma, representable functors. Direct sums and products. Direct and inverse limits. Universal objects.

- *Homological algebra.* Additive and abelian categories, additive functors. Exactness. Injective and projective objects. Derived functors. Category R–mod. Complexes, snake lemma and long exact sequences of homology. Hom and $\otimes$ functors. Ext and Tor functors.

- *Galois theory.* Algebraic extensions. Finite and perfect fields, splitting fields, algebraic closure. Normal, separable extensions (finite Galois extensions). Independence of characters. Roots of unity, cyclotomic extensions, abelian and solvable extensions. Kummer extensions. Infinite Galois extensions; Krull topology.

Grading. The grade will be based on homework assignments (25%), one take-home exam (10%), one mid term (25%) and the final (40%).

- *Homework.* I will assign and collect homework weekly. Typically that will be a Friday, with few exceptions (see Course Schedule on page 3). You are encouraged to work together, but your write ups must be your own. **Late homework will not be graded**.

- *Exams.* There will be three exams: a take home, a midterm and the final. Dates of these exams are included in the Course Schedule (page 3). The last two will be closed-book, in-hall exams - similar to the algebra qualifying.

- *Class Participation and Attendance.* Although attendance is not regularly monitored frequent absences are likely to be noted and may factor into the grade in borderline cases.

Prerequisites. We will keep the prerequisites to a minimum. Abstract Algebra I (6111) is an official prerequisite for this course. However we will go over the background material as needed.

Suggested texts

The course will be based entirely on the lecture notes. If you miss a class (not recommended) you can download the notes at the following

https://people.math.osu.edu/gautam.42/S18/notes.html

There are a lot of excellent textbooks on abstract algebra. The following list of recommended texts is to provide some (optional) reading suggestions, and more examples/problems to work out.

- H. Cartan and S. Eilenberg, *Homological algebra*, Princeton Landmarks in Mathematics.
- S. Lang, *Algebra*, 3rd edition, Springer.
- N. Bourbaki, *Algebra Chapter 5 : commutative fields*, Springer.
- N. Bourbaki, *Algebra Chapter 10: homological algebra*, Springer.
- N. Jacobson, *Basic Algebra I, II*, Dover.
- E. Artin *Galois theory*, Dover.
- A. Grothendieck, *Sur quelques points d'algèbre homologique*, Tôhoku Math. J. (1957).

General Policies

Academic Misconduct. It is the responsibility of the Committee on Academic Misconduct to investigate or establish procedures for the investigation of all reported cases of student academic misconduct. The term "academic misconduct includes all forms of student academic misconduct wherever committed; illustrated by, but not limited to, cases of plagiarism and dishonest practices in connection with examinations. Instructors shall report all instances of alleged academic misconduct to the committee (Faculty Rule 3335-5-487). For additional information, see the Code of Student Conduct (http://studentaffairs.osu.edu/info_for_students/csc.asp).

Disability Services. Students with disabilities that have been certified by the Office for Disability Services will be appropriately accommodated and should inform the instructor as soon as possible of their needs. The Office for Disability Services is located in 150 Pomerene Hall, 1760 Neil Avenue; telephone 292-3307, TDD 292-0901; http://www.ods.ohio-state.edu/

Course Schedule

The following schedule is tentative only. You will be notified of any changes by email, or in class. The most recent version of this syllabus will remain available at
https://people.math.osu.edu/gautam.42/S18/syllabus.pdf

Week	Topics	Homework deadline
1 1/8-12	Definitions: categories, injective and surjective morphisms. Functors. Natural transformations. Equivalence of categories.	HW 1 due on 1/19
January 15 - Martin Luther King Jr. day, no class		
2 1/17-19	Faithful, full functors. Adjoint functors. Yoneda's lemma. Representability.	HW 2 due on 1/26
3 1/22-26	Direct sums and direct products. Direct and inverse limits.	HW 3 due on 2/2
4 1/29-2/2	Additive categories. Kernel and cokernel. Abelian categories. Additive functors. Exactness (left/right) of functors.	HW 4 due on 2/9
5 2/5-9	Injective and projective objects. Generalities on sequence of functors (derived functors). Tensor product. Flat modules.	No homework
Take home exam assigned on February 12. Due on February 13.		
6 2/12-16	Category of complexes. (Co)Homology. Connecting morphisms. Snake lemma. Quasi-isomorphisms and homotopy.	HW 5 due on 2/23
7 2/19-23	Baer's criterion for injectivity. Existence of injective modules. Projective and injective resolutions. Uniqueness.	HW 6 due on 3/2
8 2/26-3/2	Ext functor. Computations using projective and injective resolutions. Relation to extensions and group cohomology.	HW 7 due on 3/9
9 3/5-9	Flat modules again. Tor functor. Computations and adjointness.	HW 8 due on 3/19 **Monday**
March 12-16 - Spring break, no classes. Mid Term on March 20 Tuesday during recitation.		
10 3/19-23	Fields. Extensions - finite, algebraic, transcendental. Splitting fields: existence and uniqueness.	HW 9 due on 3/30
11 3/26-30	Group characters. Galois group of an extension. Normal extensions. Separability. Galois extensions. Fundamental Theorem of Galois theory.	HW 10 due on 4/6
12 4/2-6	Finite fields. Perfect fields. Differential criterion for separability. Cyclotomic fields. Abelian and solvable extensions.	HW 11 due on 4/13
13 4/9-13	Noether's equations. Kummer extensions. Algebraic closure. Separability and normality for infinite extensions.	HW 12 due on 4/20
14 4/16-20	Topological groups. Profinite groups. Galois group of infinite extensions. Krull topology. Compactness of Galois group.	No homework
April 23 - Last day of classes		

Final exam: Friday April 27 12-1.45PM. Location TBD.