COMPLEX ANALYSIS: PRACTICE EXAM 1

Problem 1. Sketch the following domain and state whether it is open/closed/neither, connected or not, bounded or not (**no justification needed**).

$$S = \left\{ z \in \mathbb{C} \text{ such that } \operatorname{Im}(z) > 1 \text{ and } \frac{\pi}{4} \leq \arg(z) \leq \frac{3\pi}{4} \right\}$$

Problem 2. Compute the limits of the following functions, as $z \to 0$.

(a) $\dfrac{e^{z^2} - 1}{z}$ (b) $\dfrac{\sin(3z)}{2z}$ (c) $\dfrac{z^2 + z - \mathbf{i}}{2z + 3}$

Problem 3. For the following function, verify the Cauchy–Riemann equations, and compute its derivative. Here, $x = \operatorname{Re}(z)$ and $y = \operatorname{Im}(z)$.

$$f(z) = e^{-y}\left((x+1)\cos(x) - y\sin(x)\right) + \mathbf{i}e^{-y}\left((x+1)\sin(x) + y\cos(x)\right)$$

Problem 4. Identify the domain of each of the functions below, and compute their derivative:

(a) $\ln(3z - 2)$. (b) $(\mathbf{i}z + 1)^{\frac{1}{3}}$.

Problem 5. Compute the real and imaginary parts of the following:

$$\left(1 + \sqrt{3}\mathbf{i}\right)^{3\pi\mathbf{i}}$$

Problem 6. Write the definition of $\cos(z), \sin(z)$ for $z \in \mathbb{C}$. Use this to verify the following identity:

$$\sin(2z)\sin(z) = \cos(z)(1 - \cos(2z)).$$

Problem 7. Compute all $z \in \mathbb{C}$ such that $z^6 + 1 = 0$. Let us call them $\alpha_1, \alpha_2, \ldots, \alpha_6$. Verify that $\alpha_1 \cdot \alpha_2 \cdot \ldots \cdot \alpha_6 = 1$.