STUDY GUIDE FOR THE FINAL

1. LIST OF TOPICS AND REFERENCES

The following topics were covered during our course. They are divided into four parts, roughly according to the material covered in mid terms.

1.1. Complex plane and $\mathbb{C}$–differentiable functions. *Lectures 0–10.*

Definition of complex numbers. Addition, multiplication, modulus, argument and complex conjugation. Triangle inequality. Open, closed, bounded and connected subsets of $\mathbb{C}$. $\mathbb{C}$–differentiability. Cauchy–Riemann equations. Examples of $\mathbb{C}$–differentiable functions (polynomial and rational functions, exponential, sine and cosine). n^{th} roots of a complex number.

Please read a summary of these topics, given in Review I:

https://people.math.osu.edu/gautam.42/S20/Exams/Review1.pdf

and some problems for practice, from Practice Mid Term I:

https://people.math.osu.edu/gautam.42/S20/Exams/Practice1.pdf

1.2. Integration over paths. Cauchy's theorems. *Lectures 11-20.*

Definition of $\int_{\gamma} f(z)\,dz$. Basic properties and an important inequality. Antiderivative theorem. Cauchy's theorem. Contours, interior/exterior, counterclockwise orientation. Principle of contour deformation. Cauchy's integral formula. Liouville's theorem and the fundamental theorem of algebra. Partial fraction decomposition of rational functions.

Please read a summary of these topics, given in Review II:

`https://people.math.osu.edu/gautam.42/S20/Exams/Review2.pdf`

and some problems for practice, from Practice Mid Term II:

`https://people.math.osu.edu/gautam.42/S20/Exams/Practice2.pdf`

1.3. **Infinite series, residues and applications.** *Lectures 21-30.*
Uniform convergence of infinite series and integrals. Power series and their radius of convergence. Taylor and Laurent series. Three types of singularities. Residues and their applications to real integrals.

Study guide for mid term 3 contains a summary of these topics, and problems for practice:

`https://people.math.osu.edu/gautam.42/S20/Exams/StudyGuideMT3.pdf`

1.4. **Special functions.** *Lectures 31-35.*
Euler's gamma function. Jacobi's theta function. Doubly–periodic functions.

Problem set 8, part III, contains a summary of Euler's Γ–function.

`https://people.math.osu.edu/gautam.42/S20/Homework/ProblemSheet8.pdf`

(Solutions are available at our course homepage, under Homework link).

For doubly–periodic functions, and Jacobi's theta function, see:

`https://people.math.osu.edu/gautam.42/S20/Homework/Elliptic.pdf`

2. About the final

The final exam will consist of 10 problems. Roughly divided as:

(1) 2 problems on the following topics: trigonometric identities, n^{th} roots of a complex number, computing real and imaginary parts of α^β for two complex numbers α, β.

See for example, Problems 5,6,7 of Mid Term 1.

(2) 2 problems about using Cauchy's integral formula, and principle of contour deformation. These problems will be about computing contour integrals, partial fractions and residues.

See Problem Set 5, problems 6–10, 13,14. Problem Set 7, problem 1.

(3) 2 problems about Taylor/Laurent series. Three types of singularities.

See Problem Set 6, problems 10–13. Study Guide for Mid term 3: sample problems 1–3, 5.

(4) 1 problem about computing real integrals.

See Problem Set 7, problems 7–10. Problem Set 8, problems 1–3.

(5) 3 problems about Γ–function, doubly–periodic functions and θ–function.

See Problem Set 8, problems 7–11. Problems about doubly–periodic functions and theta function: problems 3,4,6,7.