ABSTRACT ALGEBRA II - MATH 6112

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COURSE INFORMATION

Homepage. https://people.math.osu.edu/gautam.42/S21/AII/AlgebraII.html

Class time. MWF 11.30AM-12.25PM.
Recitations. TR 11.30AM-12.25PM.
Both lectures and recitations will be online, via CarmenZoom. You should be able to connect to Zoom with audio, video and chat participation.

Course description. This is the second semester of the PhD level abstract algebra sequence. It is now part of the qualifying exam system. To receive credit for the course as part of the qualifying exam structure, you must obtain an A- or better.

Contents. The following topics are considered as the syllabus for the qualifying exam.

- *Category theory.* Categories, functors, natural transformations. Equivalence of categories. Yoneda's lemma, representable functors. Direct sums and products. Direct and inverse limits. Universal objects.

- *Homological algebra.* Additive and abelian categories, additive functors. Exactness. Injective and projective objects. Derived functors. Category R–mod. Complexes, snake lemma and long exact sequences of homology. Hom and $\otimes$ functors. Ext and Tor functors.

- *Galois theory.* Algebraic extensions. Finite and perfect fields, splitting fields, algebraic closure. Normal, separable extensions (finite Galois extensions). Independence of characters. Roots of unity, cyclotomic extensions, abelian and solvable extensions. Kummer extensions. Infinite Galois extensions; Krull topology.

Grading. The grade will be based on homework assignments (20%), three mid term exams (20% each) and the final (20%).

- *Homework.* Homework is an essential component of this course. Its goal is to help you understand the material as well as to develop mathematical skills. It is thus imperative that you start working on it as soon as it is assigned, and seek help (from the instructors and your classmates) if you are stuck on a problem.

    Homework will be assigned on a weekly basis, due on Fridays as indicated on the course schedule below. Assignments will be submitted electronically via Carmen in pdf format (typeset in LaTeX). Each homework will be graded for correctness and clarity of presentation. The best solutions will be posted (with approval by each author) on the course's website. **Late homework will not be accepted without medical excuse**.

    You are strongly encouraged to discuss the problems with me, Prof. Kiers and your classmates, but your write ups must be your own. If you use other people's ideas, including from an online source, you must state this explicitly. Failure to do so might be considered as an act of plagiarism under the Academic Misconduct policy (see below). Active participation on Carmen discussion forum will

contribute towards your homework grade.

- *Exams.* There will be a total of four exams: three midterms and the final. Dates of these exams are included in the Course Schedule.

- *Class Participation and Attendance.* Although this course will be conducted entirely online, it is important to stay actively engaged with the material and connected with both instructors and class-mates, e.g. by using Carmen's discussion board.

  Online lectures and recitations will be approached as active learning sessions, in particular, through discussions in small groups. Lectures will be recorded to accommodate special situations, but I expect students to attend the lectures while they are being delivered. Frequent absences are likely to be noted and may factor into the grade in borderline cases.

Prerequisites. We will keep the prerequisites to a minimum. Abstract Algebra I (6111) is an official prerequisite for this course. However we will go over the background material as needed.

Suggested texts

The course will be based entirely on the lecture notes. Recordings of the online lectures, as well as the lecture notes will be made available on Carmen, and at:

https://people.math.osu.edu/gautam.42/S21/AII/notes.html

There are a lot of excellent textbooks on abstract algebra. The following list of recommended texts is to provide some (optional) reading suggestions, and more examples/problems to work out.

- H. Cartan and S. Eilenberg, *Homological algebra*, Princeton Landmarks in Mathematics.

- S. Lang, *Algebra*, 3$^{\text{rd}}$ edition, Springer.

- N. Bourbaki, *Algebra Chapter 5 : commutative fields*, Springer.

- N. Bourbaki, *Algebra Chapter 10: homological algebra*, Springer.

- N. Jacobson, *Basic Algebra I, II*, Dover.

- E. Artin *Galois theory*, Dover.

- A. Grothendieck, *Sur quelques points d'algèbre homologique*, Tôhoku Math. J. (1957).

Course Schedule

The following schedule is tentative only. You will be notified of any changes by email or Carmen announcements. The most recent version of this syllabus will remain available at
https://people.math.osu.edu/gautam.42/S21/AII/syllabus.pdf

Week	Topics	Homework
1 1/11-15	Definitions: categories, injective and surjective morphisms. Functors. Natural transformations. Equivalence of categories.	HW 1 due on 1/22
January 18 - Martin Luther King Jr. day, no class		
2 1/18-22	Faithful, full functors. Adjoint functors. Yoneda's lemma. Representability.	HW 2 due on 1/29
3 1/25-29	Direct sums and direct products. Direct and inverse limits.	HW 3 due on 2/5
4 2/1-5	Additive categories. Kernel and cokernel. Abelian categories. Additive functors. Exactness (left/right) of functors.	No homework
Mid term 1. February 8 Monday.		
5 2/8-12	Injective and projective objects. Generalities on sequence of functors (derived functors).	HW 4 due on 2/19
6 2/15-19	Category of complexes. (Co)Homology. Connecting morphisms. Snake lemma. Quasi-isomorphisms and homotopy.	HW 5 due on 2/26
Instructional break. February 23,24. No class.		
7 2/22-26	Category R–mod, Baer's criterion for injectivity. Projective and injective resolutions. Uniqueness.	HW 6 due on 3/5
8 3/1-5	Ext and Tor functors. Computations.	HW 7 due on 3/12
9 3/8-12	Flat modules. Tor functor again. Computations and adjointness.	No homework
Mid term 2. March 15 Monday.		
10 3/15-19	Fields. Extensions - finite, algebraic, transcendental. Splitting fields: existence and uniqueness.	HW 8 due on 3/26
11 3/22-26	Group characters. Galois group of an extension. Galois extensions. Fundamental Theorem of Galois theory.	HW 9 due on 4/2
Instructional break. March 31, April 1. No class		
12 3/29-4/2	Finite fields. Perfect fields. Differential criterion for separability. Primitive element theorem.	HW 10 due on 4/9
13 4/5-9	Noether's equations. Hilbert's 90th problem. Galois cohomology. Application to cyclic extensions.	No homework
Mid term 3. April 12 Monday.		
14 4/12-16	Kummer extensions. Topological groups. Profinite groups. Krull topology.	HW 11 due on 4/23
15 4/19-23	Fundamental theorem of infinite Galois extensions. Compactness of the Galois group.	No homework
April 23 - Last day of classes		
Final exam: Wednesday April 28.		

Course technology

The course will be delivered entirely online. You should be able to connect to CarmenZoom with audio, video and chat participation. Course annoncements will be made regularly through Carmen. **It is strongly encouraged that you connect to Carmen regularly (at least three times a week).**

For help with your password, university e-mail, Carmen, or any other technology issues, questions, or requests, contact the OSU IT Service Desk. Standard support hours are available at

<p align="center">https://ocio.osu.edu/help/hours.</p>

Support for urgent issues is available 24x7.

- *Self-Service and Chat support:* http://ocio.osu.edu/selfservice
- *Phone:* 614-688-HELP (4357)
- *Email:* 8help@osu.edu
- *TDD:* 614-688-8743

General policies

Academic Misconduct Statement: It is the responsibility of the Committee on Academic Misconduct to investigate or establish procedures for the investigation of all reported cases of student academic misconduct. The term "academic misconduct" includes all forms of student academic misconduct wherever committed; illustrated by, but not limited to, cases of plagiarism and dishonest practices in connection with examinations. Instructors shall report all instances of alleged academic misconduct to the committee (Faculty Rule 3335-5-48.7). For additional information, see the Code of Student Conduct at http://studentlife.osu.edu/csc/.

Statement on Title IX: Title IX makes it clear that violence and harassment based on sex and gender are Civil Rights offenses subject to the same kinds of accountability and the same kinds of support applied to offenses against other protected categories (e.g., race). If you or someone you know has been sexually harassed or assaulted, you may find the appropriate resources at http://titleix.osu.edu or by contacting the Ohio State Title IX Coordinator, Kellie Brennan, at titleix@osu.edu

Disability Statement: The University strives to make all learning experiences as accessible as possible. If you anticipate or experience academic barriers based on your disability (including mental health, chronic or temporary medical conditions), please let me know immediately so that we can privately discuss options. To establish reasonable accommodations, I may request that you register with Student Life Disability Services. After registration, make arrangements with me as soon as possible to discuss your accommodations so that they may be implemented in a timely fashion. SLDS contact information: slds@osu.edu; 614-292-3307; http://www.ods.osu.edu/; 098 Baker Hall, 113 W. 12th Avenue.

Your mental health: As a student you may experience a range of issues that can cause barriers to learning such as strained relationships, increased anxiety, alcohol/drug problems, feeling down, difficulty concentrating and/or lack of motivation. These mental health concerns or stressful events may lead to diminished academic performance or reduce a student's ability to participate in daily activities. The Ohio State University offers services to assist you with addressing these and other concerns you may be experiencing. If you or someone you know are suffering from any of the aforementioned conditions, you can learn more about the broad range of confidential mental health services available on campus via the Office of Student Life's Counseling and Consultation Service (CCS) by visiting https://ccs.osu.edu or calling 614- 292-5766. CCS is located on the 4th Floor of the Younkin Success Center and 10th Floor of Lincoln Tower. You can reach an on-call counselor when CCS is closed at 614-292-5766. Emergency help is also available through the 24/7 National Suicide Prevention Hotline at 1-800-273- TALK or at https://suicidepreventionlifeline.org