

MATH 5522H: COMPLEX ANALYSIS

Instructor. Sachin Gautam. Office MW 640. gautam.42@osu.edu

Office hours. Wednesdays 3-4PM. Or, by appointment.

Zoom information. Available on Carmen. This class is scheduled to meet in-person. This Zoom meeting id/passcode will be used in the following events.

- University decides to suspend in-person classes. Or, the instructor gets sick (you will be notified via e-mail/carmen).
- If you get sick, please do not come to in-person class, or office hours. Just let me know and we can schedule to meet online and go over the material covered in class.

COURSE INFORMATION

Homepage. <https://people.math.osu.edu/gautam.42/S22/CA/complex.html>
<https://carmen.osu.edu>

Class time and place. MTWRF 10.20-11.15AM. University Hall 047.

Textbook. This course will primarily be based on the lectures. I will upload the lecture notes, periodically, at the following link, as well as on Carmen. So, in case you miss a class (**not recommended**) you can download the notes.

<https://people.math.osu.edu/gautam.42/S22/CA/notes.html>

There are several excellent undergraduate-level textbooks on Complex Analysis. Topics listed in the course schedule below can be found in the following recommended texts.

- (1) *Complex Analysis* by J. Bak and D.L. Newman, undergraduate texts in mathematics, Springer (2010).
- (2) *An introduction to complex function theory* by Bruce P. Palka, undergraduate texts in mathematics, Springer (1991).

In addition, I will assign problems from the following “problem book”.

A collection of problems on complex analysis, L.I. Volkovyskii, G.L. Lunts and I.G. Aramanovich, Dover publications revised edition (2011).

Contents. This is a one-semester introductory course on complex analysis. Our focus will be on methods and applications of complex function theory and contour integration. The course will often follow the “definitions → examples → theorems → applications” style. We will go over almost all the proofs of the results we encounter, but the real emphasis will be kept on problem-solving, which will also form the basis on which you will be evaluated.

Grading. Your overall grade will be determined by:

- Homework 15%.

- Three midterm exams 20% each.
- Final exam 25%.

Homework. I will assign homework every week on Friday, and will be due the next Friday (see Course Schedule on page 3 for details). It will be available on Carmen and at:

<https://people.math.osu.edu/gautam.42/S22/CA/homework.html>

Homework is an integral component of the course, and absolutely indispensable in order to get a good grade, as well as understand the material. If you are having trouble with the homework, it will be essential to seek help: ask questions in class, come to office hours, discuss with friends and so on. You are encouraged to work together on homework problems, but your write-ups must be your own. **Late homework will not be accepted.**

Homework submissions: You will be submitting your homework assignments through Carmen, and **in pdf format**. This document can be generated in two ways which will not cause major disruptions to what you have been doing for past courses. For those of you who have access to a tablet, this should be simple (e.g. by using Notability on an Ipad). Otherwise, you can scan your handwritten solutions with your smartphone and generate a pdf. There are several apps and tutorials online for this, see for example:

<https://edu.gcfglobal.org/en/mobile-device-tips/how-to-scan-documents-with-a-smartphone/1/>

GENERAL POLICIES

Academic Misconduct. It is the responsibility of the Committee on Academic Misconduct to investigate or establish procedures for the investigation of all reported cases of student academic misconduct. The term “academic misconduct” includes all forms of student academic misconduct wherever committed; illustrated by, but not limited to, cases of plagiarism and dishonest practices in connection with examinations. Instructors shall report all instances of alleged academic misconduct to the committee (Faculty Rule 3335-5-487). For additional information, see the Code of Student Conduct (http://studentaffairs.osu.edu/info_for_students/csc.asp).

Disability Services. Students with disabilities that have been certified by the Office for Disability Services will be appropriately accommodated and should inform the instructor as soon as possible of their needs. The Office for Disability Services is located in 150 Pomerene Hall, 1760 Neil Avenue; telephone 292-3307, TDD 292-0901; <http://www.ods.ohio-state.edu/>

COURSE SCHEDULE

The following schedule is tentative only. You will be notified of any changes by email, or in class. The most recent version of this syllabus will remain available at <https://people.math.osu.edu/gautam.42/S22/CA/syllabus.pdf>

| Week | Topics | HW |
|---|---|------------------|
| 1 1/10-14 | Complex numbers. Algebraic operations. Polar form. n^{th} roots. e^z , $\sin(z)$, $\cos(z)$ and $\log(z)$. Topology of complex plane. | HW1 Due 1/21 |
| January 17 - Martin Luther King day, no class | | |
| 2 1/17-21 | Functions of a complex variable. Limits, continuity. Point at ∞ Riemann sphere. \mathbb{C} -differentiability. Cauchy-Riemann equations. | HW2 Due 1/28 |
| 3 1/24-28 | CR equations cntd. Harmonic functions. Line integrals. Cauchy's theorem. Cauchy's integral formula. | HW3 Due 2/4 |
| 4 1/31-2/4 | Applications: deformation of contours, Liouville's theorem. Fundamental theorem of algebra. Partial fractions. | No HW |
| Mid term 1 on Tuesday February 8. | | |
| 5 2/7-11 | Review I. Mid term I. Applications cntd: max modulus principle Schwarz' lemma. Automorphisms of unit disc. Poisson integral. | HW4 Due 2/18 |
| 6 2/14-18 | Applications cntd. Evaluation of real integrals. Jordan's lemma. Cauchy's principal value. Sequence and series. Uniform convergence. | HW5 Due 2/25 |
| 7 2/21-25 | Weierstrass' theorem. Power series. Radius of convergence. Taylor/Laurent series. Poles/essential singularities. Residue. | HW6 Due 3/4 |
| 8 2/28-3/4 | Applications. Hurwitz, Rouché and Casorati-Weierstrass thms. Automorphisms of plane and sphere. Möbius transformations. | No HW |
| Mid term 2 on Tuesday March 8. | | |
| 9 3/7-11 | Review II. Mid term II. Mittag-Leffler problem. Weierstrass' infinite product expansions. | HW7 Due 3/25 |
| Spring break - March 14-18. | | |
| 10 3/21-25 | Functions defined by integrals. Laplace transform. Euler's gamma function. | HW8 Due 4/1 |
| 11 3/28-4/1 | Conformal mappings. Riemann mapping theorem. Dirichlet's principle. Examples. | HW9 Due 4/8 |
| 12 4/4-8 | Schwarz-Christoffel transformation. Elliptic integrals. Doubly-periodic functions (basic theory). | No HW |
| Mid term 3 on Tuesday April 12. | | |
| 13 4/11-15 | Review III. Mid term III. Jacobi's theta function. Elliptic sine and cosine functions. | HW10 Due 4/22 |
| 14 4/18-22 | Applications of elliptic functions. | No HW |
| Last day of classes April 25 Monday | | |
| Final exam: April 28 Thursday 10-11.45AM | | |