

APPLIED COMPLEX VARIABLES II (MATH 7652)

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COURSE INFORMATION

Homepage. <https://people.math.osu.edu/gautam.42/Sp24/ACV/complex.html>

Class time and place. 9.10am - 10.05am, Enarson 206.

Office hours. MWF 10.30-12.

Homework and grading. Your grade will be determined by your participation in the class, during office hours, and (possibly) a final presentation. I will compile a list of problems:

<https://people.math.osu.edu/gautam.42/Sp24/ACV/Problems.pdf>

Class participations means that you should attempt and discuss (some of) these problems with me, during office hours, and your classmates.

Textbook. The course will be based on lecture notes, which will be regularly uploaded at the following link: <https://people.math.osu.edu/gautam.42/Sp24/ACV/notes.html>

For reference, I will be using the following texts.

- M.J. Ablowitz and A.S. Fokas, *Complex variables: introduction and applications*.
- O. Costin, *Asymptotics and Borel summability*.
- R. Courant, *Dirichlet's principle, conformal mapping and minimal surfaces*.
- D. Hilbert and R. Courant, *Methods of mathematical physics, volume I*.
- E.T. Whittaker and G.N. Watson, *A course of modern analysis*.

CONTENTS

This course is the second of a year-long sequence of two courses. It is aimed at giving a thorough treatment of various applications of complex analysis, to conformal geometry, asymptotic analysis, special functions and systems of difference, differential and integral equations.

Part 1. *Summability of a series.*

- Various methods of resummation: Euler, Abel, Cesàro and Borel.
- Review of Laplace transform, Watson's lemma, and method of steepest descent. Stokes' phenomenon. Examples of differential and difference equations. WKB method.
- Examples of resurgent series and alien (*étrange*) calculus.
- Approximating sequences. Padé convergents and continued fractions.

Part 2. *Integral transforms and integral equations*

- Riemann–Hilbert factorization problem. Plemelj formula.
- Integral operators and (singular) integral equations. Fredholm theory.
- Dirichlet series and Mellin transform.

Part 3. *Elliptic integrals.*

- Arc length of ellipse, arithmetic–geometric means, Poncelet's porism for circles.
- Jacobi's elliptic sine and cosine functions. Weierstrass \wp function. $SL_2(\mathbb{Z})$ -action on the upper half plane. Modular forms.

Part 4. *Riemann surfaces.*

- Basics. Covering spaces. Riemann surface associated to germs of holomorphic functions and forms.
- Uniformization theorem and applications.

GENERAL POLICIES

Academic Misconduct. It is the responsibility of the Committee on Academic Misconduct to investigate or establish procedures for the investigation of all reported cases of student academic misconduct. The term “academic misconduct” includes all forms of student academic misconduct wherever committed; illustrated by, but not limited to, cases of plagiarism and dishonest practices in connection with examinations. Instructors shall report all instances of alleged academic misconduct to the committee (Faculty Rule 3335-5-487). For additional information, see the Code of Student Conduct (http://studentaffairs.osu.edu/info_for_students/csc.asp).

Disability Services. Students with disabilities that have been certified by the Office for Disability Services will be appropriately accommodated and should inform the instructor as soon as possible of their needs. The Office for Disability Services is located in 150 Pomerene Hall, 1760 Neil Avenue; telephone 292-3307, TDD 292-0901; <http://www.ods.ohio-state.edu/>