ACCELERATION-INDUCED CARRIER [a] OF THE IMPRINTS OF GRAVITATION

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We exhibit a purely quantum mechanical carrier of the imprints of gravitation by identifying for a relativistic charge a property which (i) is independent of its mass and (ii) expresses the Poincare invariance of spacetime in the absence of gravitation. This carrier is a Klein-Gordon-equation-determined vector field given by the "Planckian power" and the "r.m.s. thermal fluctuation" spectra.

Does there exist a purely quantum mechanical carrier of the imprints of gravitation? The motivation for considering this question arises from the following historical scenario: Suppose the time is 1907 when Einstein had the "happiest thought of his life", which launched him on the path toward his formulation of gravitation (general relativity). But suppose Einstein already knew relativistic quantum mechanics, and that, in fact, he accepted and appreciated it without any reservations before he started on his journey. How different would his theory of gravitation have been from what we have today? Put differently, how different would the course of history have been if Einstein had grafted relativistic quantum mechanics onto the *roots* of gravitation instead of its trunk or branches?

Nontrivial relativistic quantum mechanics starts with the Klein-Gordon equation

$$\frac{\partial^2\psi}{\partial t^2} - \frac{\partial^2\psi}{\partial z^2} + k^2\psi = 0, \tag{1}$$

where $k^2 = k_x^2 + k_y^2 + m^2$.

The objective of this brief report is to deduce from this equation a carrier of the imprints of gravitation with the following three fundamental requirements:

1. The imprints must be carried by the evolving dynamics of a quantum mechanical wavefunction.

2. Even though the dynamical system is characterized by its particle mass m, the carrier and imprints must *not* depend on the particle species, i.e. the carrier must be *independent* of k^2. This requirement is analogous to the classical one in which the world line of a particle is independent of its mass.

3. In the absence of gravitation the carrier should yield measurable results (expectation values) which are invariant under Lorentz boosts and spacetime translations.

In quantum mechanics the wave function plays the role which in Newtonian mechanics is played by a particle trajectory or in relativistic mechanics by a particle world line. That the wave function should also assume the task of carrying the imprints of gravitation is, therefore, a reasonable requirement.

Because of the Dicke-Eotvos experiment, the motion of bodies in a gravitational field is independent of the composition of these bodies. Consequently, the motion of

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free particles in spacetime traces out particle histories whose details depend only on the gravitational environment of these particles, not on their internal constitution. The superposition of different wave functions (states) of a relativistic particle yields interference fringes which do depend on the mass of a particle. If the task of these wave functions is to serve as carriers of the imprints of gravitation, then, unlike in classical mechanics, these interfering wave functions would do a poor job at their task: They would respond to the presence (or absence) of gravitation in a way which depends on the details of the internal composition (mass) of a particle. This would violate the simplicity implied by the Dicke-Eotvos experiment. Thus we shall not consider such carriers. This eliminates any quantum mechanical framework based on energy and momentum eigenfunctions because the dispersion relation, $E^2 = m^2 + p_z^2 + p_y^2 + p_x^2$, of these waves depends on the internal mass m.

Recall that momentum and energy are constants of motion which imply the existence of a locally inertial reference frame. Consequently, requirement 2. rules out inertial frames as a viable spacetime framework to accomodate any quantum mechanical carrier of the imprints of gravitation. Requirement 2. also rules out a proposal to use the interference fringes of the gravitational Bohm-Aharanov effect to carry the imprints of gravitation [1]. This is because the fringe spacing depends on the rest mass of the quantum mechanical particle.

Requirement 3. expresses the fact that the quantum mechanical carrier must remain unchanged under the symmetry transformations which characterize a two-dimensinal spacetime. By overtly suppressing the remaining two spatial dimensions we are ignoring the requisite rotational symmetry. Steps towards remedying this neglect have been taken elsewhere [2].

We shall now exhibit a carrier which fulfills the three fundamental requirements. That carrier resides in the space of Klein-Gordon solutions whose spacetime domain is that of a *pair* of frames accelerating into opposite directions ("Rindler frames"). These frames partition spacetime into a pair of isometric and achronally related Rindler Sectors I and II,

$$\left.\begin{array}{l} t - t_0 = \pm\xi \sinh\tau \\ z - z_0 = \pm\xi \cosh\tau \end{array}\right\} \quad \begin{array}{ll} + : & \text{"Rindler Sector I"} \\ - : & \text{"Rindler Sector II"} \end{array} \tag{2}$$

Suppose we represent an arbitrary solution to the K-G equation in the form of a complex two-component vector normal mode expansion

$$\begin{pmatrix} \psi_I(\tau,\xi) \\ \psi_{II}(\tau,\xi) \end{pmatrix} = \int_{-\infty}^{\infty} \{ a_\omega \begin{pmatrix} 1 \\ 0 \end{pmatrix} + b_\omega^* \begin{pmatrix} 0 \\ 1 \end{pmatrix} \} \sqrt{2|\sinh\pi\omega|} \frac{K_{i\omega}(k\xi)}{\pi} e^{-i\omega\tau} d\omega \equiv \int_{-\infty}^{\infty} \psi_\omega d\omega \tag{3}$$

This is a *correlated* ("entangled") state with two degrees of freedom. Besides the continuum of boost energies, this state has a *discrete polarization* degree of freedom. Its two components refer to the wave amplitude at diametrically opposite events on a Cauchy hypersurface $\tau = constant$ in Rindler I and II respectively.

This representation puts us at an important mathematical juncture: We shall forego the usual picture of viewing this solution as an element of Hilbert space with the usual Klein-Gordon inner product. Instead, we shall adopt a much more powerful viewpoint based on the vector bundle $C^2 \times R$. Here C^2 is the complex vector space of two-spinors, which is the fiber over the one-dimensional base manifold $R = \{\omega : \ -\infty < \omega < \infty\}$, the real line of Rindler frequencies in the mode integral, Eq.(3).

We know that one can add vectors in the *same* vector (fiber) space. However, one may not, in general, add vectors belonging to different vector spaces at different ω's. The exception is when vectors in different vector spaces are *parallel*. In that case one may add these vectors. The superposition of modes, Eq.(3), demands that one do precisely that in order to obtain the two respective total amplitudes of Eq.(3).

The mode representation of Eq.(3) determines two parallel spinor fields over R, one corresponding to "spin up", the other to "spin down". It is not difficult to verify that these two spinor fields are (Klein-Gordon) orthonormal in each fiber over R. The spinor field

$$\{\left(\begin{array}{c} a_\omega \\ b^*_\omega \end{array}\right) : \ -\infty < \omega < \infty\} \quad . \tag{4}$$

is a section of the fiber bundle $C^2 \times R$ and it represents a linear combination of the two parallel vector fields. It is clear that there is a *one-to-one correspondence between* $\Gamma(C^2 \times R)$*, the* ∞*-dimensional space of sections of this spinor bundle, and the space of solutions to the Klein-Gordon equation.* Our proposal is to have each spinor field serve as a carrier of the imprints of gravitation: A gravitational disturbance confined to, say, Rindler I or II would leave its imprint on a spinor field at $\tau = -\infty$ by changing it into another spinor field at $\tau = +\infty$.

We know that in the absence of gravitation each of the positive and negative Minkowski plane wave solutions evolves independently of all the others. This scenario does not change under Lorentz boosts and spacetime translations. Will the proposed carriers comply with this invariance, which is stipulated by fundamental requirement 3.? To find out, consider a typical plane wave, which in the spinor representation (3) is a state with a high degree of correlation between the boost energy and the polarization ("spin") degrees of freedom. Suppose for each boost energy we determine the normalized Stokes parameters of this polarization, i.e. the three Klein-Gordon based expectation values of the "spin" operator $\vec{\sigma}/2$. This is a three-dimensional vector field over the base manifold R, and is given by [2]

$$\frac{\langle \psi_\omega, \vec{\sigma} \psi_{\omega'} \rangle}{\langle \psi_\omega, \psi_{\omega'} \rangle} = \pm \left(\sqrt{N(N+1)}, 0, \frac{1}{2} + N \right); \quad N = (e^{2\pi\omega} - 1)^{-1}; \quad -\infty < \omega < \infty$$

In compliance with requirements 2. and 3., this vector field is (a) *independent of the particle mass* and (b) the same for *all* positive (negative) Minkowski plane wave modes, a fact which expresses its Poincare invariance. The presence of *gravitation would leave its imprints by producing characteristic alterations in this vector field.*

Its obvious but noteworthy feature is that its components coincide with the "Planckian power" and the "r.m.s. thermal fluctuation" spectra, in spite of the fact that we are only considering the quantum mechanics of a single charge.

References

[1] J.S.Anandan in B.L.Hu, M.P.Ryan, and C.V.Vishveshwara (eds.), *Directions in General Relativity, Volume 1*, (Cambridge University Press, 1993) p.10

[2] U.H.Gerlach in R.T.Jantzen and G.M.Keiser (eds.), *The Seventh Marcel Grossmann Meeting, Part B*, World Scientific Publishing Co. (1996), *ibid* International Jour. of Mod. Phys. 11, 3667 (1996) p.957