## Syllabus and Reading Assignments for Math 603.02

### Prerequisites:

- By the 2nd week know how to solve (a) a second order ordinary differential equation with constant coefficients, (b) Euler's homogeneous differential equation.

- By the 3rd week know (a) the **CAUCHY-GOURSAT theorem**, (b) the RESIDUE of a complex analytic function, (c) **CAUCHY'S INTEGRAL FORMULA**.

(Sections and chapters refer to "Linear Mathematics in Infinite Dimensions", March 2011 Beta Edition)

### Week 1

**Monday**
- Section 1.5.7: "Isomorphic Hilbert Spaces"
  - You should already know Section 1.5.6 and Theorem 1.5.2

**Wednesday**
- Chapter 2: "Why Fourier Theory?"
  - Section 2.1: "The Dirichlet Kernel"
  - Section 2.1.2: "Fraunhofer-Kirchoff diffraction theorem"

**Friday**
- Section 2.1.2: "Sampling theorem"
  - Fourier series theorem"

### Week 2

**Monday**
- In Section 2.1.2, pages 56-58: "Poisson's summation formula"
  - Section 2.2: "The Dirac Delta Function"
  - Section 2.3: "The Fourier Integral" (Transition from Fourier Series to Fourier Integral)

**Wednesday**
- Section 2.3: "The Fourier Integral" (The Fourier Integral Theorem)
  - Fourier Transform as a Unitary Transformation"

**Friday**
- In Section 2.3: "Fourier Transform via Parseval's Relation"
  - Supplementary handout: "Wavelets and Multiresolution analysis"
  - Section 2.4: "Orthonormal Wave Packet Representation"
  - Section 2.4.1: "Construction"
  - Section 2.4.2: "Definition and Properties"

**Reminder:**
- (i) Know the **Cauchy-Goursat Theorem**

### Week 3

**Monday**
- Section 2.4.3: "Phase Space Representation"

**Wednesday**
- Section 2.4.3: "Phase Space Representation"
  - Orthornormal Wavelet Representation"

**Friday**
- Handout: "Wavelets and Multiresolution analysis"
  - Section 2.5: "Orthonormal Wavelet Representation"

**Reminder:**
- (ii) Know how to solve Euler's differential equation
  - (iii) Know Cauchy's integral theorem

### Week 4

**Monday**
- Section 2.6.7: "The Pyramid Algorithm"

**Wednesday**
- Section 4.1: "The Adjoint of an Operator"
  - Section 4.1.1: "Adjoint Boundary Conditions"
  - Section 4.1.2: "Second Order Operator and the Bilinear Concomitant"

**Friday**
- Section 4.1: "The Adjoint of an Operator"
  - Section 4.2: "Green's Function and its Adjoint"

**Reminder:**
- (i) Know how to solve Euler's differential equation
  - (ii) Know the **Cauchy Goursat theorem**
  - (iii) Know **Cauchy's integral theorem**
  - (iv) Know how to find the Residue of a simple pole

### Week 5

**Monday**
- Section 4.3: "Pictorial Definition of a Green's Function"
  - Section 4.3.1: "The Simple String and Poisson's Equation"
  - Section 4.3.2: "Point Force Applied to the System"

**Wednesday**
- Section 4.3.2: "Properties and Utility of Green's function", "The Fundamental Theorem for Green's Functions"

**Friday**
- Section 4.6: "Unit Impulse Response: General Homogeneous Boundary conditions"
  - Theorem 4.4.2: "Uniqueness of a Green's Function" (review)
  - Section 4.8.2: "Spectral Resolution of the Green's Function"

**Reminder about prerequisites:**
- From Math 602 you should already know **Section 3.3.3**, namely:
  - Basic Properties of a Sturm-Liouville Eigenvalue Problem"

### Week 6

**Monday**
- Section 4.11: "Spectral Representation"
  - Section 4.11.2: "Contour Integration around a Branch Cut"

**Wednesday**
- Section 4.11.3: "Fourier Sine Theorem"
  - Start reading Chapter 5: "Special Function Theory"

The purpose of this chapter is to develop a most powerful method for putting the nature of waves and their propagation into a mathematically tractable form. Pages 279-280 are reminders from linear algebra (Math 601 and 602). The powerful method is illustrated by means of the Helmholtz equation. Thus you should grasp:

- Sections 5.1.1: "Cartesian vs. Polar coordinates"
- Sections 5.1.2: "Degenerate Eigenvalues"
- Sections 5.1.3: "Complete Set of Commuting Operators"
- Sections 5.1.4: "Translations and Rotations in the Euclidean Plane"
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<td>5.7, p 355-357</td>
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