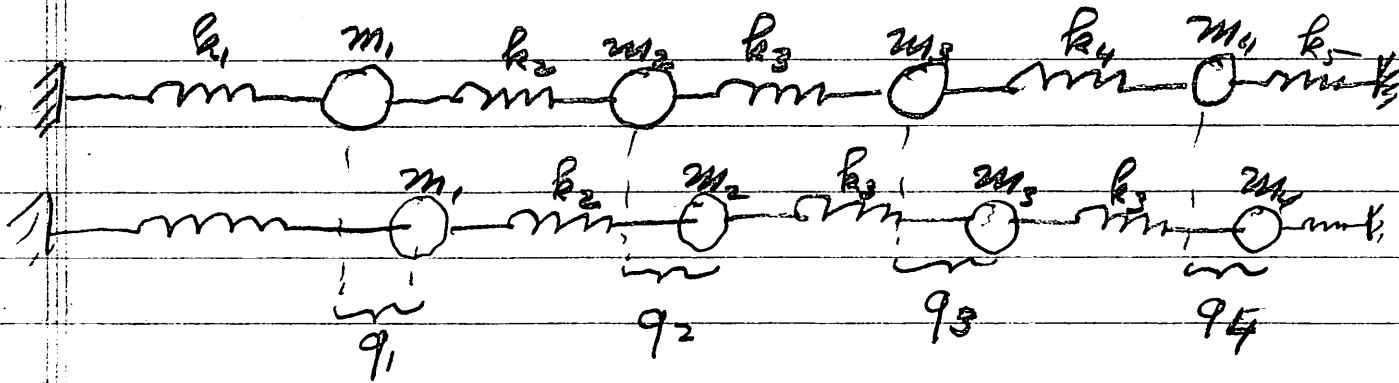


APPENDIX A (- for Lecture 39) System of Four coupled masses



q_i = amplitude of m_i away from equil'm.

Newton's 2nd Law

$$m_1 \ddot{q}_1 = k_1(0 - q_1) + k_2(q_2 - q_1) = -(k_1 + k_2)q_1 + k_2 q_2$$

$$m_2 \ddot{q}_2 = k_2(q_1 - q_2) + k_3(q_3 - q_2) = k_2 q_1 - (k_2 + k_3)q_2 + k_3 q_3$$

$$m_3 \ddot{q}_3 = k_3(q_2 - q_3) + k_4(q_4 - q_3) = k_3 q_2 - (k_3 + k_4)q_3 + k_4 q_4$$

$$m_4 \ddot{q}_4 = k_4(q_3 - q_4) + k_5(0 - q_4) = k_4 q_3 - (k_4 + k_5)q_4$$

or

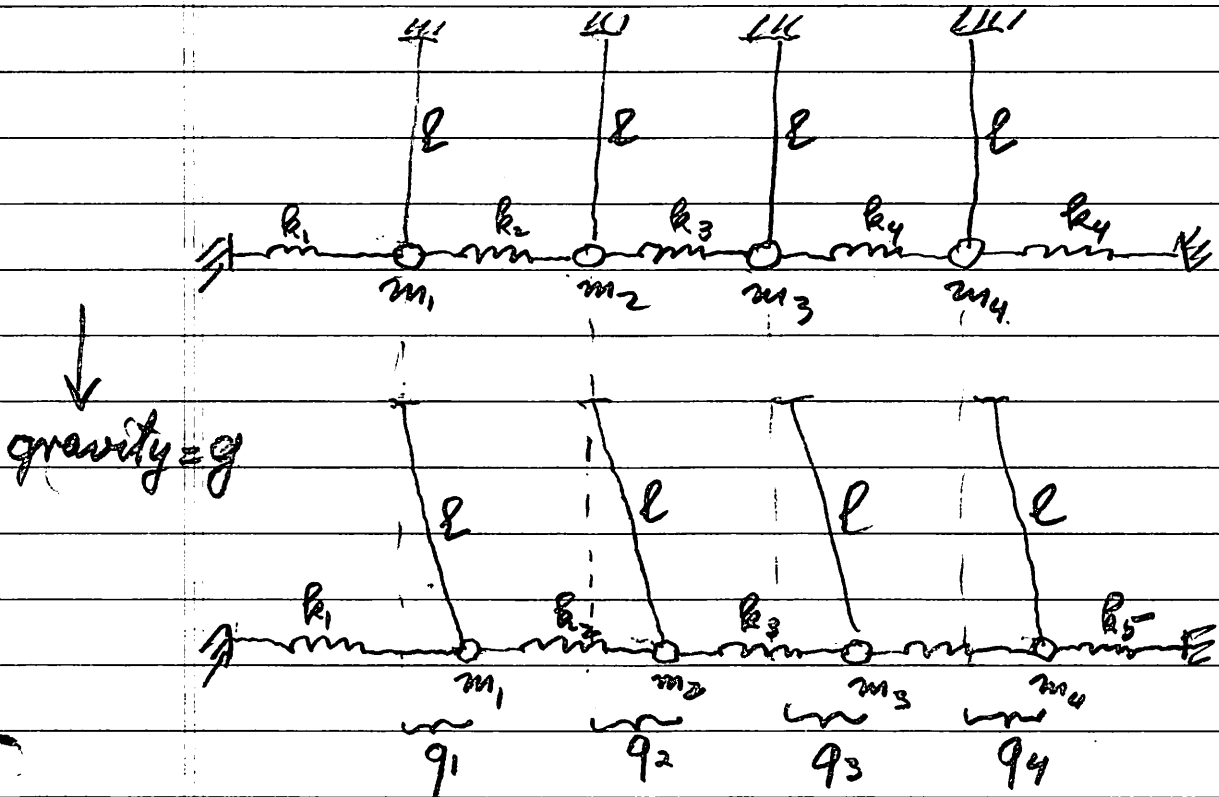
$$0 = \begin{bmatrix} m_1 & & & \\ & m_2 & & \\ & & m_3 & \\ & & & m_4 \end{bmatrix} \frac{d^2}{dt^2} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} + \begin{bmatrix} +(k_1 + k_2) & -k_2 & 0 & 0 \\ -k_2 & +(k_2 + k_3) & -k_3 & 0 \\ 0 & -k_3 & +(k_3 + k_4) & -k_4 \\ 0 & 0 & -k_4 & +(k_4 + k_5) \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}$$

or

$$\underline{B} \ddot{\underline{q}} + \underline{A} \underline{q} = 0$$

where positive definite symmetric

System of Four coupled pendulums.



q_i = amplitude away from equilibrium
 Newton's 2nd Law

$$m_1 \ddot{q}_1 = -m_1 g \frac{q_1}{l} - k_1 (q_1 - 0) + k_2 (q_2 - q_1)$$

$$m_2 \ddot{q}_2 = -m_2 g \frac{q_2}{l} - k_2 (q_2 - q_1) + k_3 (q_3 - q_2)$$

$$m_3 \ddot{q}_3 = -m_3 g \frac{q_3}{l} - k_3 (q_3 - q_2) + k_4 (q_4 - q_3)$$

$$m_4 \ddot{q}_4 = -m_4 g \frac{q_4}{l} - k_4 (q_4 - q_3) + k_5 (0 - q_4)$$

$$\begin{matrix} m_1 \\ m_2 \\ m_3 \\ 0 \\ m_4 \end{matrix} \begin{bmatrix} \frac{d^2}{dt^2} \\ \frac{d^2}{dt^2} \\ \frac{d^2}{dt^2} \\ \frac{d^2}{dt^2} \\ \frac{d^2}{dt^2} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} + \begin{bmatrix} -m_1 \frac{g}{l} + (k_1 + k_2) & -k_2 & 0 & 0 \\ -k_2 & -m_2 \frac{g}{l} + (k_2 + k_3) & -k_3 & 0 \\ 0 & -k_3 & -m_3 \frac{g}{l} + (k_3 + k_4) & -k_4 \\ 0 & 0 & -k_4 & -m_4 \frac{g}{l} + k_5 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Remark: (1) When $k_i = 0$, we have a system of 4 independent pendulum (oscillators)

(2) When $L \rightarrow \infty$, we have a system of 4 coupled masses, i.e., a finite transmission line consisting of 4 lumped circuits

(3) The eq'ns of motion are

$$\underbrace{B}_{\text{positive definite}} \ddot{\vec{q}} + \underbrace{A}_{\text{symmetric}} \dot{\vec{q}} = 0$$

where B is positive definite and A is symmetric.