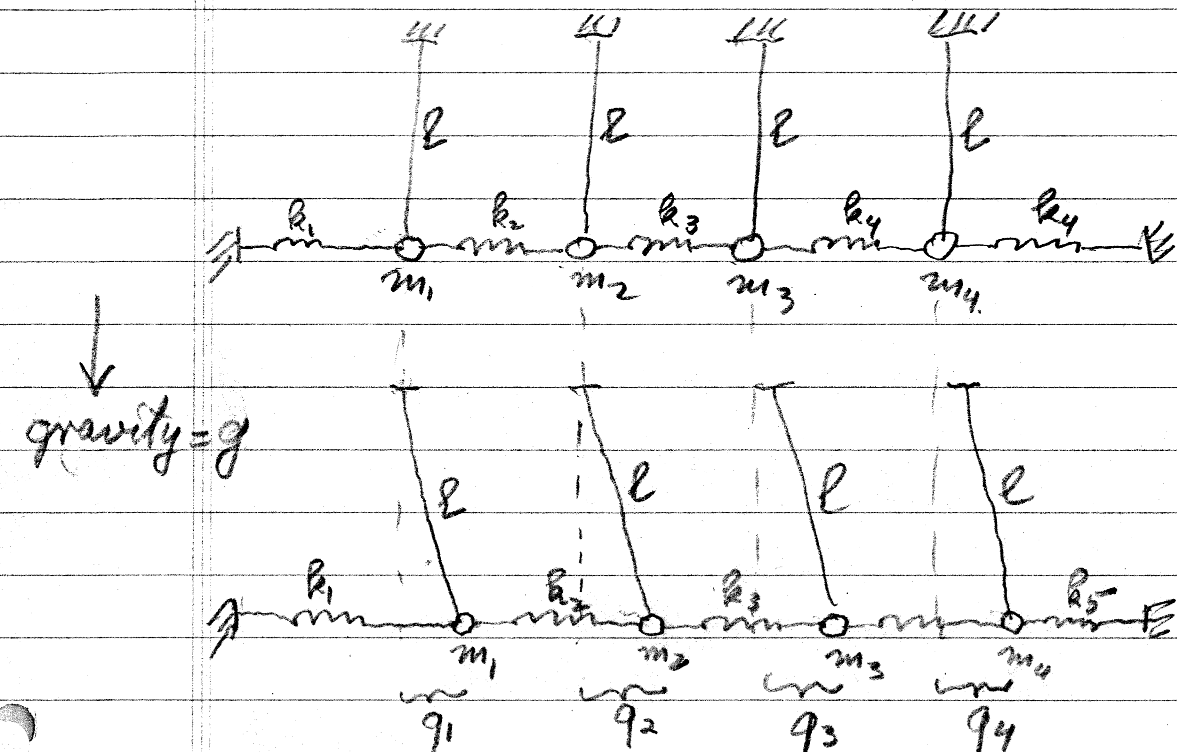
APPENDIX B (for Lecture 39)

Systems of Four coupled pendulum.



q_i = amplitude away from equilibrium

Newton's 2nd Law

$$m_1 \ddot{q}_1 = -m_1 g \frac{q_1}{l} - k_1 (q_1 - 0) + k_2 (q_2 - q_1)$$

$$m_2 \ddot{q}_2 = -m_2 g \frac{q_2}{l} - k_2 (q_2 - q_1) + k_3 (q_3 - q_2)$$

$$m_3 \ddot{q}_3 = -m_3 g \frac{q_3}{l} - k_3 (q_3 - q_2) + k_4 (q_4 - q_3)$$

$$m_4 \ddot{q}_4 = -m_4 g \frac{q_4}{l} - k_4 (q_4 - q_3) + k_5 (0 - q_5)$$

$$0 = \begin{bmatrix} m_1 & & & \\ & m_2 & & \\ & & m_3 & \\ 0 & & & m_4 \end{bmatrix} \frac{d^2}{dt^2} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} + \begin{bmatrix} m_1 \frac{g}{l} + (k_1 + k_2) & -k_2 & 0 & 0 \\ -k_2 & m_2 \frac{g}{l} + (k_2 + k_3) & -k_3 & 0 \\ 0 & -k_3 & m_3 \frac{g}{l} + (k_3 + k_4) & -k_4 & 0 \\ 0 & 0 & -k_4 & m_4 \frac{g}{l} + k_5 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}$$

Remark: (1) When $k_i = 0$, we have a system of 4 independent pendulum (oscillators)

(2) When $\ell \to \infty$, we have a system of 4 coupled masses, i.e. a finite transmission line consisting of 4 lumped circuits

(3) The eq'ns of motion are

$$\underset{\sim}{B}\,\ddot{\vec{q}} + \underset{\sim}{A}\,\vec{q} = 0$$

where positive definite symmetric.