Math 5192 **Homework 0** Due: TBD

1. If A, B, and C are constants, the differential equation

$$Ax^2X'' + BxX' + CX = 0 \tag{1}$$

is called a *Cauchy-Euler equation*.

(a) Exhibit two independent solutions to this equation.

(b) (i) Show that with the substitution $x = e^s$ this differential equation can be put into the form

$$A\frac{d^2Y}{ds^2} + (B - A)\frac{dY}{ds} + CY = 0. \tag{2}$$

(ii) Also, write down the relation between X and Y which allows the values of X to be evaluated in terms of the values of Y. (iii) Finally, write down the relation between Y and X which allows the values of Y to be evaluated in terms of those of X.
(iii) Finally, write down the relation between Y and X which allows the values of Y to be evaluated in terms of those of X. In fact, you might even consider doing (ii) and (iii) before doing (i).
(iv) When $A = B = 1$ and $C = \lambda > 0$, exhibit

(1) two independent complex solutions to Eq.(1)
(2) two independent real solutions to Eq.(1)
(3) two independent complex solutions to Eq.(2)
(4) two independent real solutions to Eq.(2)