Modern Mathematical Methods in Relativity Theory II

Tentative
Table of Contents
[and Reading List]

1.-2. World lines of extremal length as geodesics [MTW 13.4].

3. The equation for a geodesic: bridge between physics and geometry. Inertial force $\neq 0$ $\Longleftrightarrow$ coordinate frame is curvilinear; equivalence principle: gravitational force = inertial force. "geometry" = "gravitation".

4. Momentum and energy: its definition and conservation [TW 7.1-7.7].

5. The particle density 3-form [MTW Box 4.4, Box 5.2, Box 15.1.f]; particle 4-current: density and flux.

6. Stress-energy tensor: the flow of momenergy; the physical significance of its components [MTW Ch.5].

7. Conservation of momenergy; creation of momenergy: (a) expressed in physical terms; (b) expressed as an integral over the boundary [MTW Box 15.1.B] of a 4-volume.

8. Creation of momenergy: (c) expressed as a 4-volume integral: Gauss's theorem for a vector valued integral; (d) expressed as the exterior derivative of a vector valued three form.

9. Conservation laws formulated in terms of the generalized Stokes theorem [MTW 5.8, Box 5.3, Box 5.4]; Summary: Conservation of momenergy expressed at four levels of mathematical generalization [MTW Box 15.2]; Expansion of a moving volume of fluid [MTW 22.1-22.3].

10. Perfect fluid and its equation of motion as implied by momenergy conservation. Energy conservation, particle conservation, and the chemical potential. Various manifestations of $T^{\nu}_{\mu;\nu} = 0$ [MTW 22.-22.3].

11. $\partial\partial V = 0$ and the Einstein field equations [MTW Ch. 15]; examples of $\partial\partial V = 0$: div curl=0, Bianchi identities.

12. Vectorial form of Stokes' theorem: the 1-2 version. Jacobi's identity [MTW Ex. 9.12 a and c] and the infinitesimal Gauss's theorem revealed by a chipped cube.

13. Gauss's theorem as a bridge from $\partial\partial V = 0$ to the Bianchi identities.

14. Moment of rotation per volume= Einstein tensor [MTW 15.4].

15. Moment of rotation, moment of force, and the Einstein field equations.

16. Einstein's eqations $\Rightarrow$ conservation of momenergy [MTW ch. 15]; integral form of the Einstein field equations; comparison with integral formulation of Coulomb's law and Ampere's law. Spherically symmetric systems.

17. Einstein's equations for spherically symmetric configurations.

18. Geometrical and matter degrees of freedom.

19. Helmholtz's theorem.

20. Integration of Einstein's field equation via mass-energy conservation; Mass distribution determines spatial geometry. Inner geometry via imbedded surface; application to the space geometry of a spherical star [MTW 23.8].

21. Simplified Einstein field equations. Equations of hydrostatic equilibrium [MTW ch.23]; equilibrium configurations: stable vs. unstable [MTW ch. 24].

22. Hamilton-Jacobi theory and the principle of constructive interference [MTW Box 25.3]. Constructive interference $\Rightarrow$ world lines have a finite length determined by Planck's constant. Derivation of Heisenberg's indeterminacy principle.

23. Reconstruction of classical world lines from the principle of constructive interference.

24. Hamilton- Jacobi analysis of the orbits of a particle in the spacetime of a spherically symmetric vacuum configuration [MTW 25.5, Box 25.4].

25. Precession of the perihelion and the deflection of light by the sun [MTW 25.5,25.6].

26. Schwarzschild spacetime: Regular behavior of proper time, proper distance, and curvature at the Schwarzschild radius r=2M [MTW 31.2]; geometry and topology of two assymptotically flat connected regions [MTW 31.6, 31.7].

27. Schwarzschild spacetime: dynamics, causal structure near r=2M; Eddington-Finkelstein coordinates [MTW 31.4, Box 31.2]; Kruskal-Szekers coordinates [MTW 31.5].

28. Globally defined coordinate system for Schwarzschild spacetime.

29. Scalar, vector, and tensor harmonics, their behavior under parity transformation; geometrical objects on 2-D Lorentz spacetime.

30. Spherical tensor harmonics; representation of generic perturbations as "odd" and "even" geometrical objects on M^2; Coordinate induced ("gauge") perturbations of a tensor field; gauge invariant geometrical objects.

Resource Texts

1. C.W. Misner, K.S. Thorne, and J.A. Wheeler, *Gravitation*, (Freeman, San Francisco, 1973).

2. E.F. Taylor and J.A. Wheeler, *Spacetime Physics.*,SECOND EDITION (1st Edition)

3. U.H. Gerlach, *Notes on Modern Mathematical Methods in Relativity Theory II*, Unpublished, Department of Mathematics, OSU, 2012