LECTURE 1: FOUR FUNDAMETAL CONCEPTS

Agenda:

1.) Bottom-Up vs Top-Down Approach

2.) Rapid course in Relativity

I) The Principle of Relativity

$\underbrace{\text{T-W 3.1-3.3}}_{\text{Sections in 2nd edition of Taylor and Wheeler}} \quad \underbrace{(1.3)}_{\text{Section in 1st edition of Taylor \& Wheeler}}$

II) Free float (="inertial) frames

T-W 2.2-2.4 (1.2, 1.4)

Fig. 2-6 in T-W

M.T.W. Fig 1.7

III) Isotropy of Space

T-W 3.12 (1.3)

2nd edition $\rightsquigarrow$ 1st edition

IV) What is an <u>Observer</u>?

T-W 2.6 + 2.7

Lecture 1

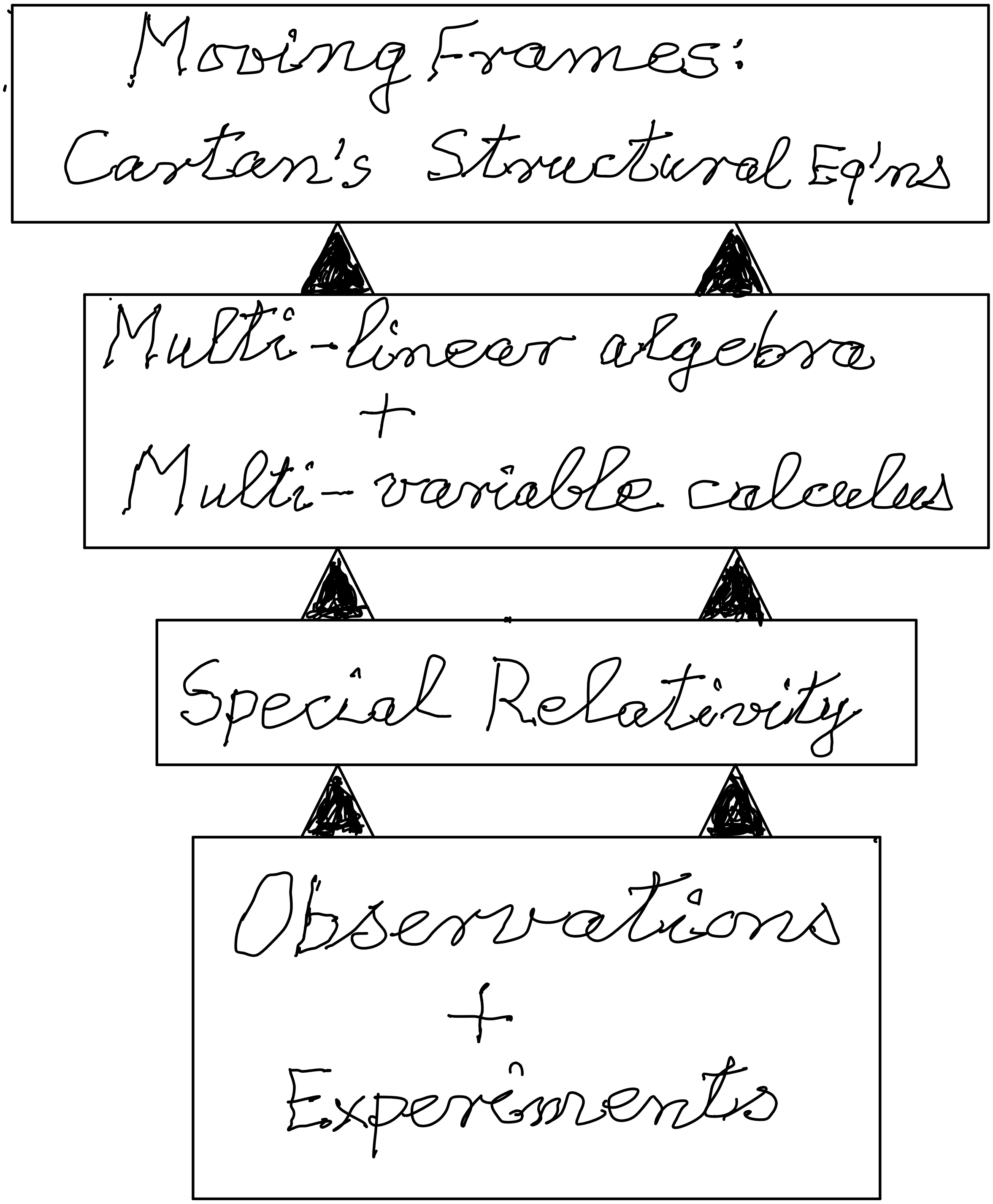
The purpose of Math 5756 is to mathematize, i.e. put into mathematical form, the laws of physics,

(
- Newton's 3 laws of motion
- Lorentz's law of motion for a charge in an e.m. field
- Maxwell's laws of electrodynamics
- Thermodynamics, etc
)

and do this in term of the modern mathematical method of the 20th century.

Our approach is that of Aristotle, the bottom up approach. It starts with the validity of the senses, i.e. with observations and experiments, and then proceeds to arrive by inductive reasoning at the laws of nature and at their applications by deductive reasoning.

For Math 5756 our edifice of knowledge is built by this bottom-up approach and thus has a multi-level structure hierarchical in abstractness.



Moving Frames:
Cartan's Structural Eq'ns

Multi-linear algebra
+
Multi-variable calculus

Special Relativity

Observations
+
Experiments

Aristotle's <u>bottom-up approach</u> is to be contrasted with Plato's <u>top-down approach</u> which starts with an arbitrary hypothesis or some intuition from a mystical Realm of Forms, and which then arrives at conclusions by purely deductive reasoning.

With Plato, knowledge is not rooted in the world, not in reality. Plato's "reality" is merely an imperfect reflection of his mystical Realm of Forms. By rejecting mysticism we go with Aristotle and shall see that the root - the fountainhead - of modern differential geometry is observations & experiment: *special relativity,* i.e. physics.

RELATIVITY

I) THE PRINCIPLE OF RELATIVITY (≡P.R.)

"All $\underbrace{\text{laws of physics}}_{(i)}$ are the same in every $\underbrace{\text{inertial reference frame}}_{(ii)}$"

The scope ("in every inertial ·· frame") and the strength of its declaration ("[applies to] all laws of physics") gives this principle universality and captures under its umbrella a diversity of implicit physical measurements.

Q: Where does the P.R. come from?

A: The P.R. is arrived at via a process of inductive reasoning applied to

(a) the observational evidence from comparisons between the outcomes of experiments performed in different inertial frames

of reference.

(b) the relevant conceptual framework

(i) Laws of physics:

- Newton's 3 Laws of motion
- Maxwell's laws of electrodynamics
- Lorentz's law of motion for a charge in an e.m. field
- Thermodynamics
- etc

The observational evidence comes from comparing the results of experiments in different inertial frames of reference:

In two such frames consider two experiments with

1. Identical instructions
2. same experimental setup
3. same procedure
4. same data set
5. same data reduction

Then, within experimental errors(!), one will observe the same result.

From this particular pair of experiments, and others like it, one infers the following generalization:

In diffent inertial frames the same experiments yield, within experimental error, the same observed results.

This finding is the same regardless of whether the two experiments involve

- Newton's 3 Laws of motion
- Maxwell's laws of electrodynamics
- Lorentz's law of motion for a charge in an e.m. field
- Thermodynamics
- or any combination such laws

This finding, therefore, applies to all laws and one has

> All laws of physics are the same in every inertial frame of reference

Thus

a) the form of these laws is the same in every inertial frame.

b) the numerical value of the physical constants $(c, \hbar, e, m, k_{\text{BOLTZ}} \dots)$ are the same w.r.t. every inertial frame

Restated negatively one has:

> "The laws of nature do not provide a way of distinguishing one inertial frame from another"

Each of these statements is called the P.R.

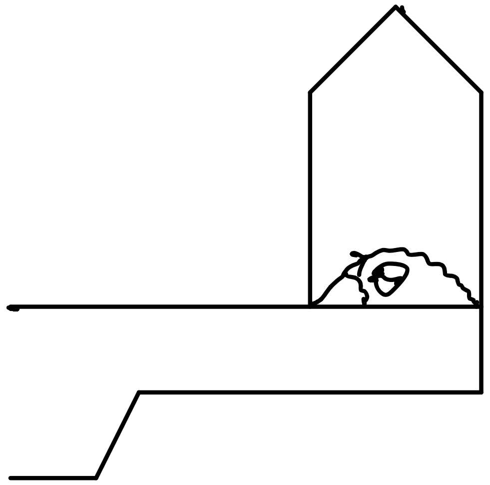
II) INERTIAL FRAMES

The relativity principle is a statement about the behaviour of things in different inertial frames.

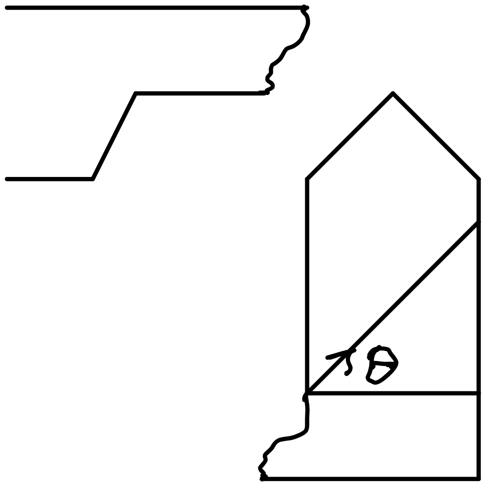
Q: How can one tell such frames from non-inertial frames?

A: Within classical (i.e. non-quantum) mechanics the answer can be given by examining the measured sharp trajectories of free particles.

Consider the following two reference frame:



Non-inertial frame



Inertial frame

An *inertial frame* is defined by Newton's 1st law of motion. (*free* particles remain at rest or in a state of *uniform*, *straight-line* motion)

1.) Definition (inertial, a.k.a. free

float frame)

A frame is said to be *inertial* (or "*free float*") to the extent that all free particles in it comply with Newton's 1st law of motion.

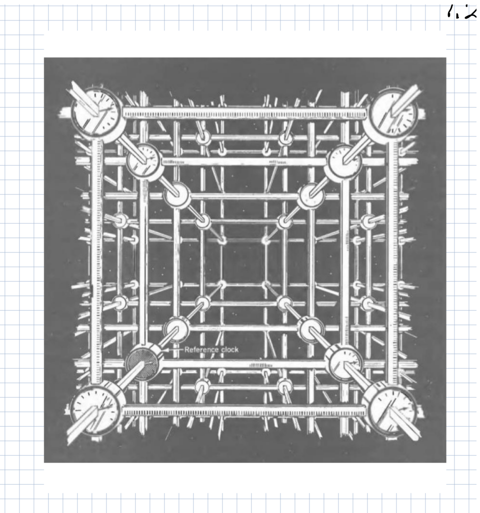
2.) More explicitly, one has the following

Definition (inertial frame, a.k.a. free float frame)

Given: (i) a region of space and an interval of time

(ii) a set of freely floating particles in this region of spacetime

Then: An inertial (= free float) frame is that region of spacetime coordinatized by a lattice work of clocks and measuring rods



Lattice work of clocks
and measuring rods

in such a way that - within some specified level of accuracy - the free particles travel

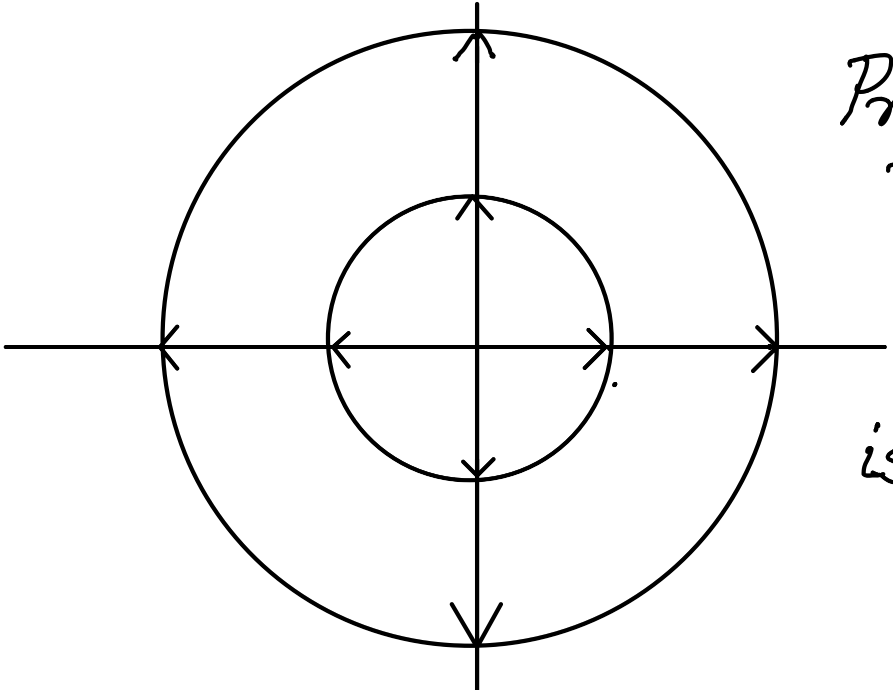
a) along straight lines
b) with constant velocity

for each and every particle
in that region of spacetime

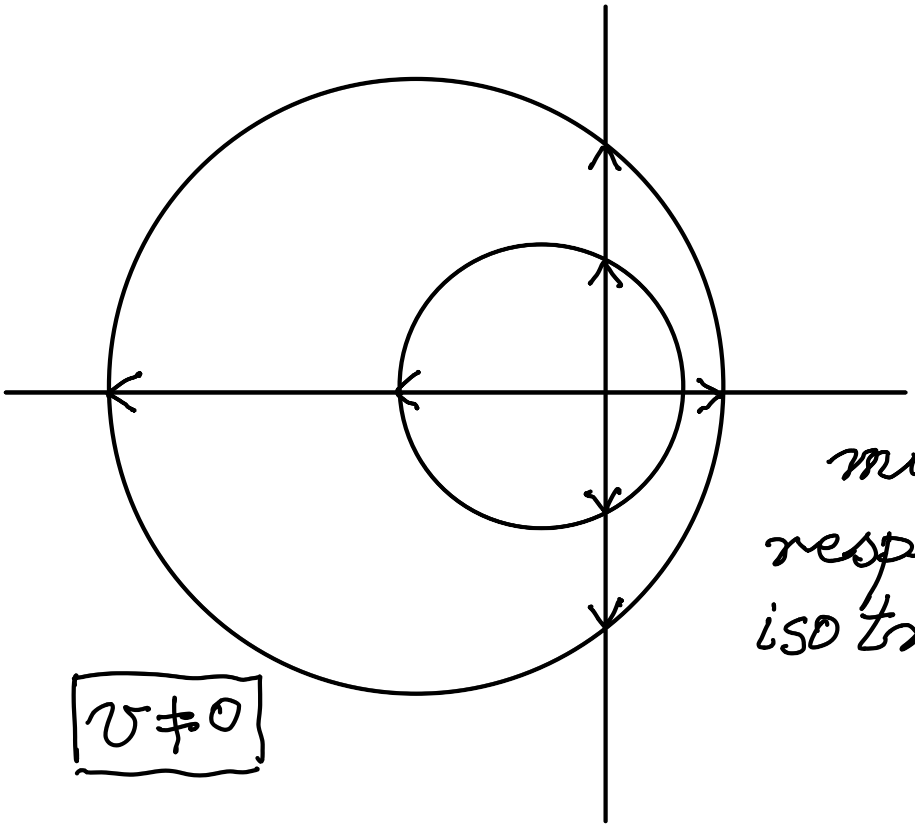
III) ISOTROPY OF SPACE

One of the surprising manifestation of the Principle of Relativity is the isotropy of light propagation in inertial frames in relative motion.

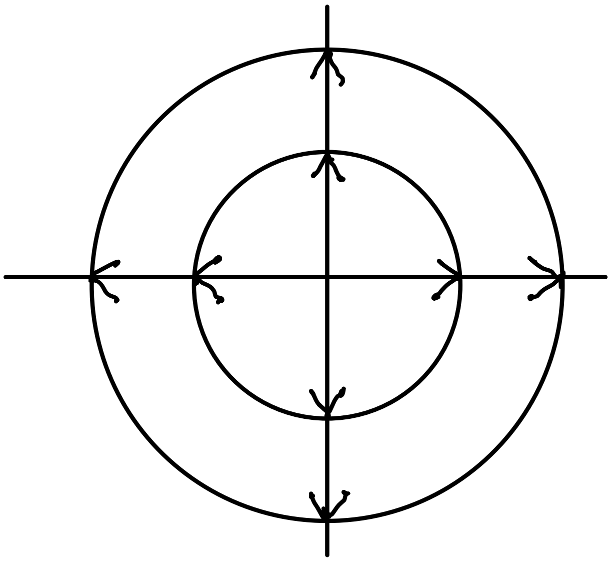
To appreciate this manifestation compare the propagation of sound with that of light in different inertial frames



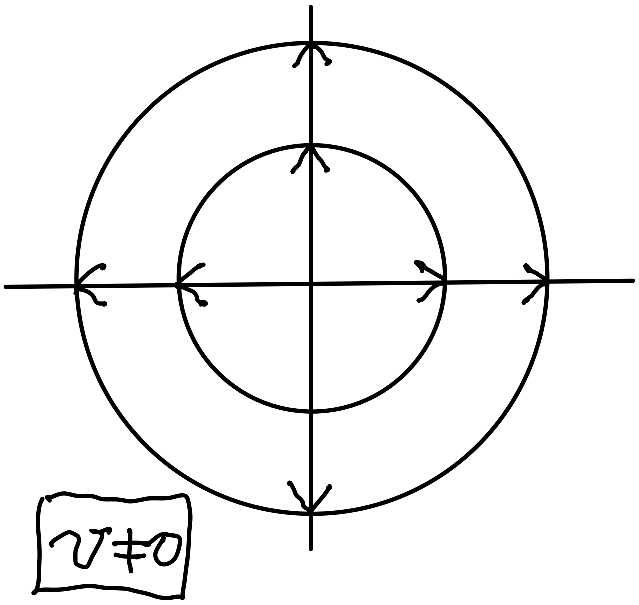
Propagating Phase fronts of sound in an isotropic medium



Propagating Phase fronts of sound in a frame moving with respect to the above isotropic medium



Propagating phase fronts of light in empty space



Propagating phase fronts of light in a frame moving w.r.t. the above frame

In a frame moving w.r.t. its medium ($v \neq 0$) sound waves propagate non-isotropically (different speeds in different directions) while e.m. waves still propagate isotropically. (same speed in all directions)

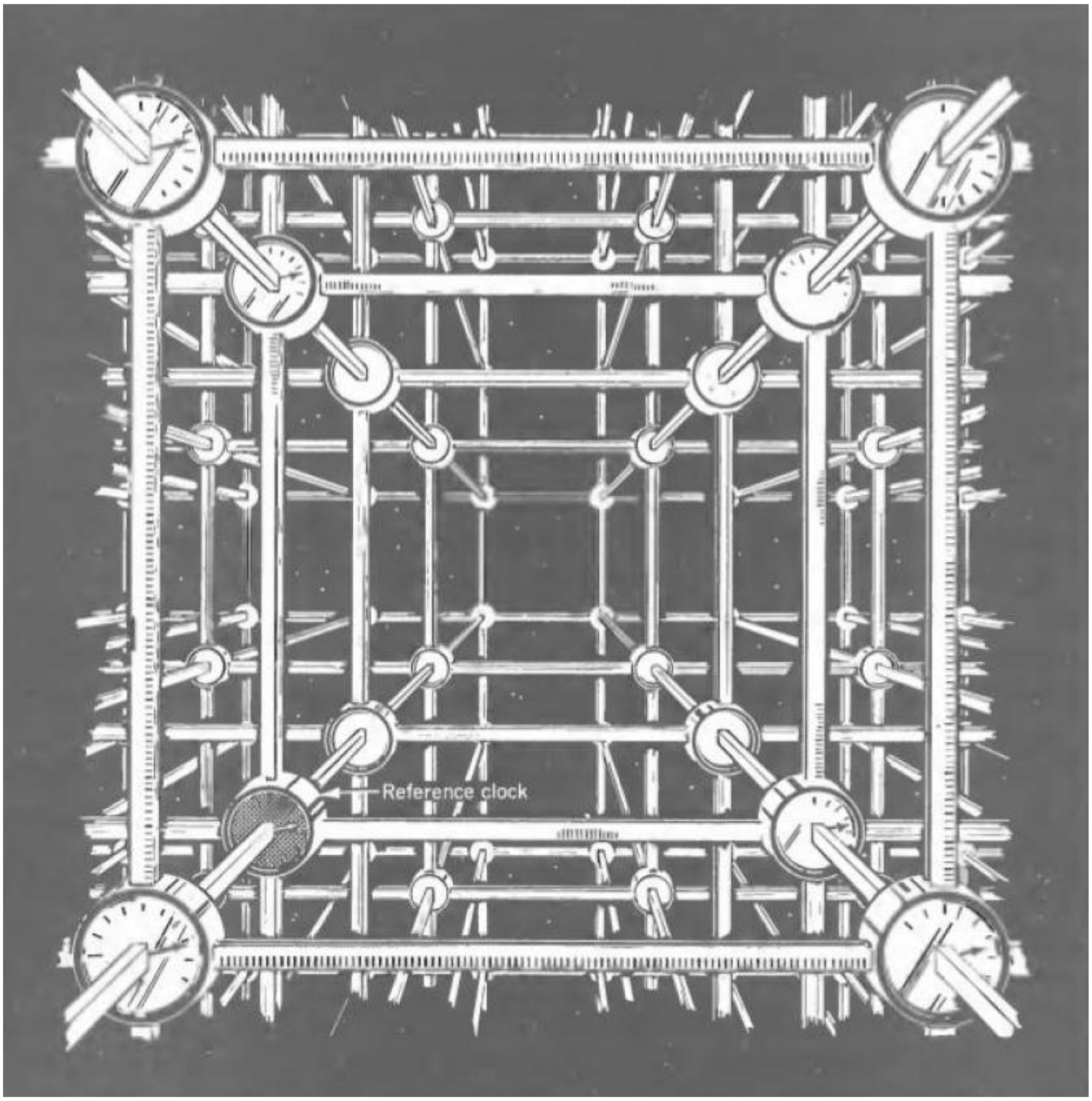
Thus one has the far-reaching result

Isotropy of space is frame independent

This principle is contained Maxwell's field equations. It also expresses the negative result of the Michelson Morley experiment

Isotropy of space says nothing about the <u>numerical value</u> of the speed of light. The Kennedy-Thorndike experiment says that also the magnitude of the velocity of light is frame independent.

Q: WHAT IS AN OBSERVER?



Latticework of clocks and measuring rods. It coordinatizes the local space time domain of an inertial reference frame as depicted above. Every such frame accommodates an agent (animate or inanimate entity) whose measurements are w.r.t. to such a lattice of clocks and rods. An observer refers to (i) such an *agent* **together with** (ii) the lattice-coordinatized *frame* surrounding that agent. Thus an observer is an agent which resides in a local frame coordinatized by clocks and measuring rods.

A: