LECTURE 7

Gravitational Red Shift

I. Doppler shift between "instantaneous Lorentz frames"

II. The Equivalence Principle:

Uniform gravity $\Longleftrightarrow$ Uniform acceleration

III. The gravitational red shift formula

Start reading Chapter 6 in MTW

I. Doppler shift between "instantaneous Lorentz frames"

Consider a bicycle travelling with uniform 4-velocity

$$\mathbf{u}: \left\{ u^0 = \frac{1}{\sqrt{1-\beta^2}},\ u^1 = \frac{\beta}{\sqrt{1-\beta^2}},\ u^2 = 0,\ u^3 = 0 \right\}$$

and a car with world line

$$\mathbb{X}(\tau): \{ t(\tau),\ x(\tau),\ 0,\ 0 \}$$

accelerating from stand-still at time $t=0$

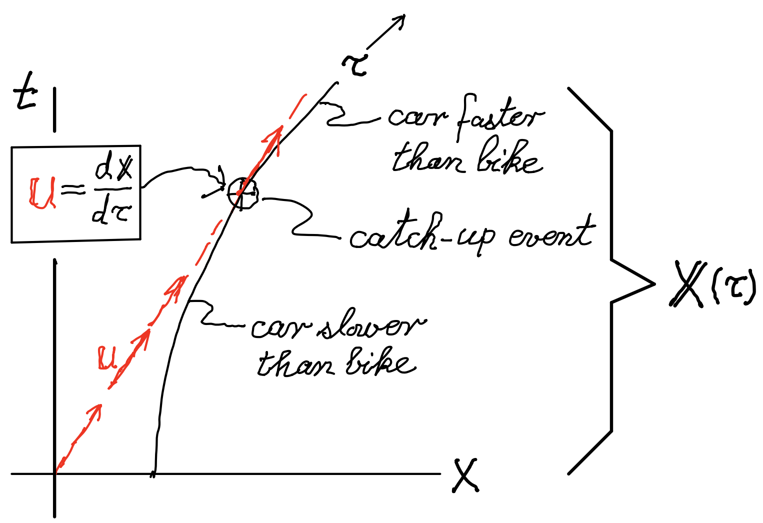


Figure 7.1: Accelerating car and inertial bicycle.

The relative velocity between the bike and the accelerating car is zero at the catch-up event.

It follows that there are two different <u>instantaneous inertial frames</u>, $S=\{(t,x)\}$ and $\bar{S}=\{(\bar{t},\bar{x})\}$

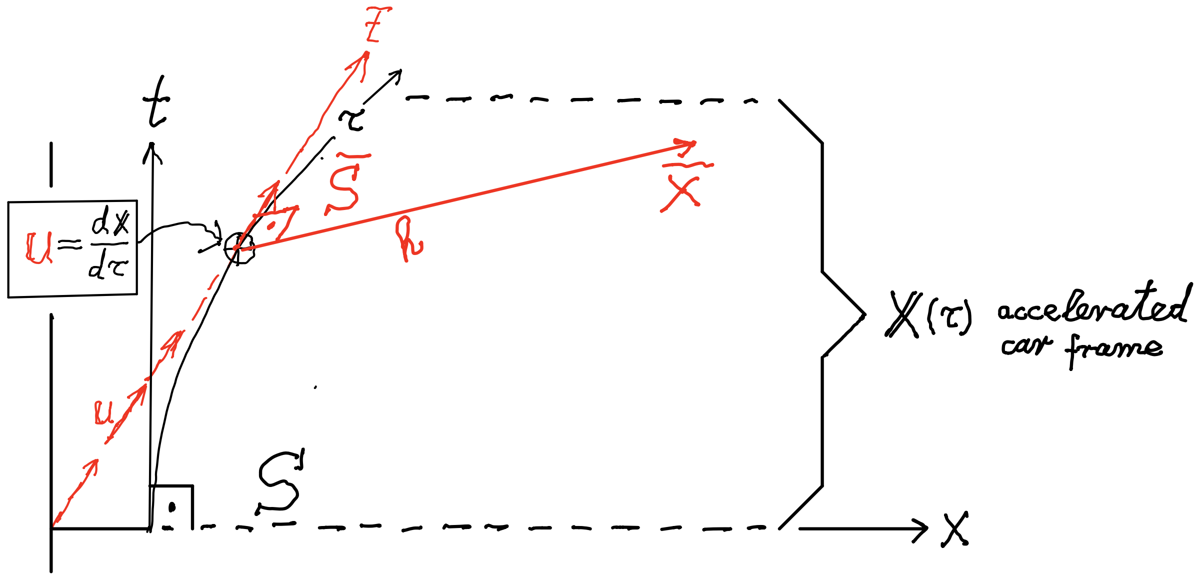


Figure 7.2: Two instantaneous Lorentz frames S and $\bar{S}$.

The interior of the car is an <u>accelerated frame of reference</u> whose world line is $X(\tau)$, and whose accelermeter measures <u>constant</u>

$$\text{acceleration} = g \quad [\text{length}/(\text{time})^2]$$

Next launch an x-ray (or laser) photon pulse of frequency ω from $x=0$ in S to $\bar{x}=h$ in $\bar{S}$.

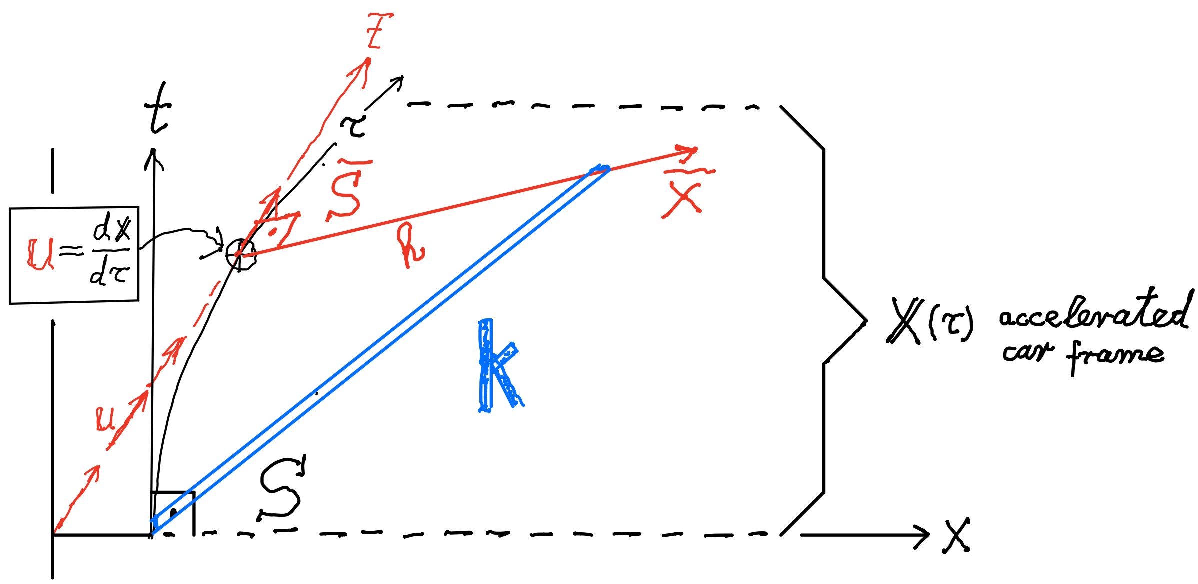


Figure 7.3: Worldline of x-ray (or laser) pulse **K**.

The time of the x-ray (or laser) pulse to transit from $x=0$ to $\bar{x}=h$ is

$$\frac{h}{c} \quad \left(\begin{array}{l}\text{"transit time"} \\ \text{in conventional units}\end{array}\right)$$

It follows that the velocity gain of $\mathcal{X}(\tau)$ during this time is

$$\frac{v}{c} = \frac{g}{c^2} h \equiv \beta .$$

This is the relative velocity between the initial frame S and the frame $\bar{S}$ of the bicycle observer when he momentarily catches up with the accelerating car $\mathcal{X}(\tau)$.

[This relative velocity is an <u>approximation</u>. It assumes that the catch-up distance of the bicycle is neglegible compared to h; it assumes that the transit time of the x-ray pulse is so short that the velocity gained by the accelerating car;

$$v = g\frac{h}{c} \ll c ,$$

is much smaller than the speed of light,

$$g\frac{h}{c^2} \equiv \beta \ll 1 .$$
]

The S-representation of the propagation 4-vector $\mathbf{k}$ is

$$\mathbf{k} : (\omega, \omega, 0, 0)$$

Its representation relative to $\bar{S}$ is

$$\mathbf{k} : (\bar{\omega}, \bar{\omega}, 0, 0),$$

where

$$\bar{\omega} = \omega \frac{1}{\sqrt{1-\beta^2}} + \omega \frac{-\beta}{\sqrt{1-\beta^2}}$$

$$\boxed{\bar{\omega} \approx \omega \left(1 - \frac{g}{c^2} h\right)} \qquad (1)$$

II. The Equivalence Principle:

Uniform gravity $\longleftrightarrow$ Uniform acceleration

The solid state forces in the protrusion X are accelerating it towards positive x relative to the two inertial frames S and $\bar{S}$ in the same way that the accelerating car in Part I was accelerating towards positive x relative to the inertial frames S of the LAB and $\bar{S}$ of the bicycle.

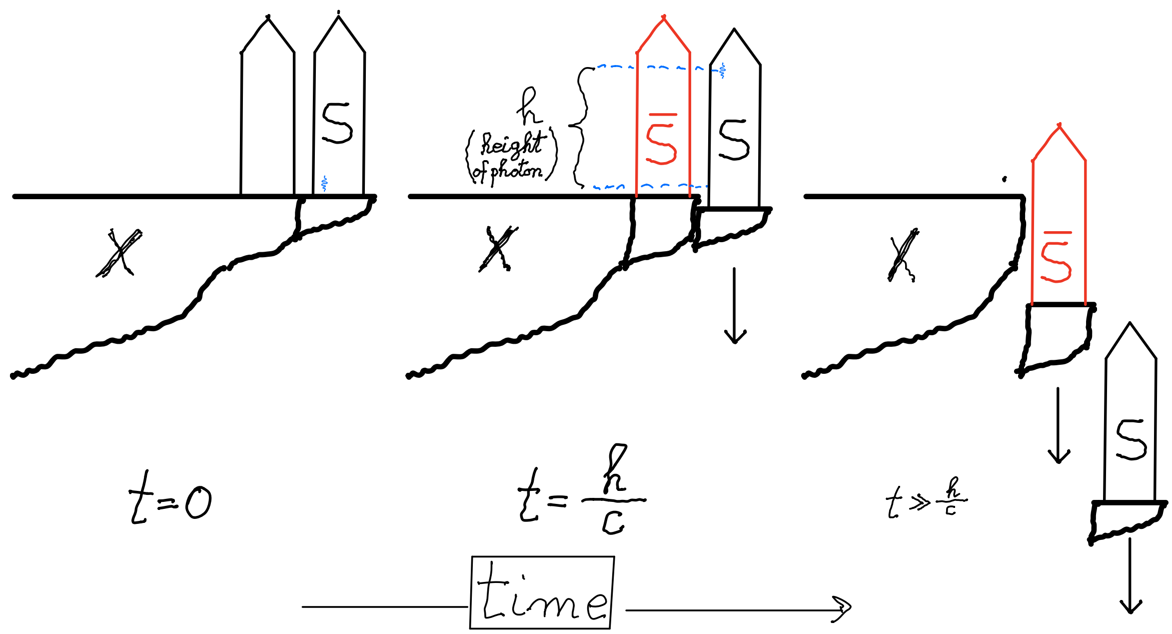


Figure 7.4: Three photographic snapshots of protrusion X accelerating relative to inertial frames S and $\bar{S}$.

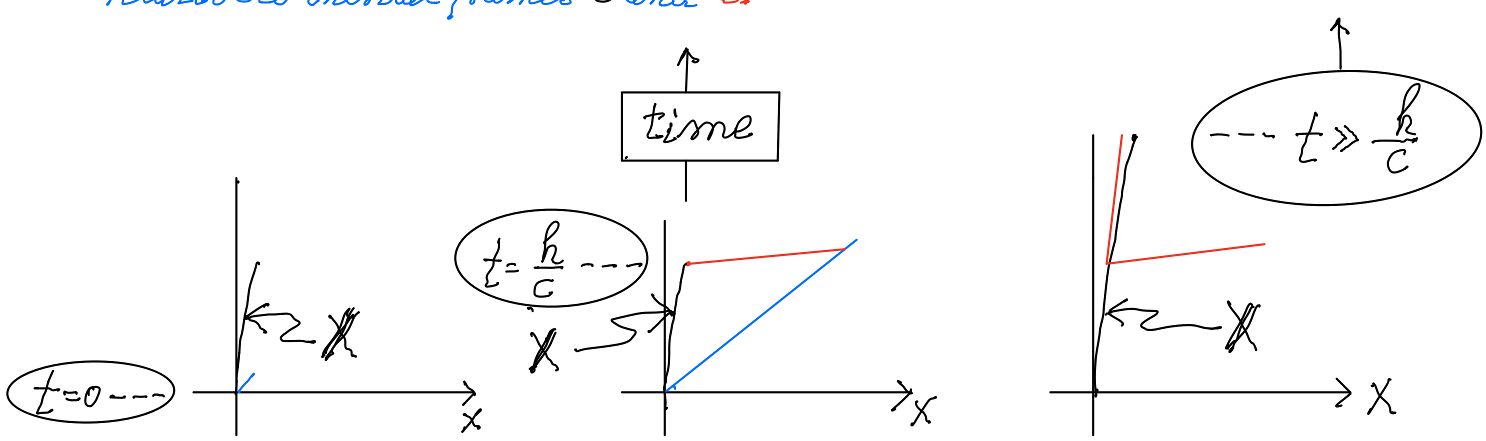
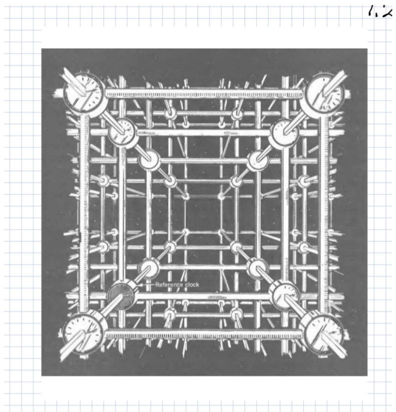


Figure 7.5: Three horizontal subspace slices (---), $t=0$, $t=\frac{h}{c}$, and $t \gg \frac{h}{c}$ corresponding to the three snapshots of Figure 7.4.

II. Gravitational redshift formula

Spacetime measurements of events along any world line are always done relative to a particular inertial frame of reference with its lattice work of clocks and rods



Lattice work of clocks
and measuring rods

which serve as a standard of measurement.

Due to gravitation, which for the earth is

$$g = 9.8 \ [\text{meter}/\text{sec}^2],$$

the protrusion X depicted in Figure 7.4 is accelerating relative to S as well as relative to $\bar{S}$. Figure 7.5 depicts this fact by means of X not being a straight worldline relative to S nor to $\bar{S}$.

This acceleration is due to the net solid state forces that act on the protrusion in order to keep it rigidly in place against the Newtonian gravitational potential, which at height h is

$$\Phi_{NEWTON} = gh$$

Applying this potential to the Doppler formula, Eq.(1) on page 7.5, one finds that

$$\boxed{\bar{\omega} = \omega\left(1 - \frac{\Phi_{NEWTON}}{c^2}\right)}$$ ("gravitational frequency dependence")

This formula mathematizes radiation changes its measured frequency in the presence of gravition. The amount of the frequency shift depends on the Newtonian potential and is given by

$$\boxed{\Delta\omega = -\frac{\Phi_{NEWTON}}{c^2}}$$ ("red shift")

Summary

The solid state forces of condensed matter physics that prevent the the earth from collapsing produce motion in each piece of the earth's crust. This motion is accelerative relative to inertial frames (ordinarily called frames in "free fall"). The accelerative world line of that motion is mathematized by a sequence of intantaneous Lorentz frames. It is the Doppler shift between them which is the gravitational redshift, which has been measured by Pound and Rebka.