(1.1)

LECTURE 1

Four Fundamental Concepts

Agenda:

1.) Metaphysics and Epistemology: Bottom-up vs. Top-Down Approach

2.) Rapid course in Relativity

I. The Principle of Relativity

T-W 3.1-3.3 (Sections in 2nd Edition of Taylor and Wheeler) (1.3) (Section in 1st Edition of Taylor and Wheeler)

II. Free Float Frames

T-W 2.2-2.4 (1.2, 1.4)

Fig. 2.6 in T-W; Fig. 1.7 in MTW

III. Isotropy of Space

T-W 3.12 (2nd Edition) (1.3) (1st Edition)

IV. What is an Observer?

T-W 2.6+2.7

(1.2(a))

The purpose of Math 5756 is to integrate linear algebra with multivariable calculus. However, before getting into the question "why", one needs to ask some meta questions about the nature of the Universe, which we deal with and which we all are part of.

- Are we in a universe which is ruled by natural laws, is stable, firm, absolute, and therefore knowable? OR
- is it an incomprehensible chaos?
- is it a realm of inexplicable miracles?
- is it an unknowable flux your mind is impotent to grasp?

Your motivation for taking Math 5756 will be different for different answers to these two types of metaphysical questions ("metaphysical" means pertaining to the nature of reality).

1.2(b)

In Math 5756-5757 we shall mathematize a number of the Universe's natural laws. They hold in anyone's spacetime environment, anywhere, anytime. With such knowledge we can know and and navigate the Universe from the local nooks and crannies here on earth to the farthest most distant galaxies.

Knowledge, any knowledge, is in the form of a hierarchical network. In the context of Math 5756 this hierarchy is as depicted in Figure 1.1.

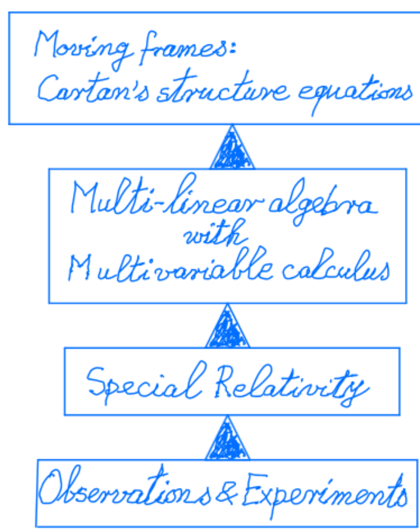

Figure 1.1: Conceptual hierarchy of to-be-developed ideas

(1.3)

This bottom-up hierarchical line of reasoning is endemic to physics and we shall adopt it in Math 5756. The fact that the starting point of all knowledge is observation was first pointed out by Aristotle (the father of logic). He directed attention to the fact that the principles of scientific knowledge are made possible and are grasped based on sense perception, i.e. observation, and it was Galileo who introduced the experimental method into physics.

Aristotle's <u>bottom-up approach</u> is to be contrasted with Plato's <u>top-down approach</u>, which starts with an arbitrary hypothesis or some "intuition" from his mystical Realm of Forms, and from which one then arrives at conclusions by purely deductive reasoning.

With Plato, knowledge is not rooted in the world, not in reality. Plato's "reality" is merely an imperfect reflection of his mystical Realm of Forms. By rejecting mysticism we go with Aristotle and shall see that the root of modern differential geometry is observation & experiment: Special Relativity, i.e. physics.

RELATIVITY

(1.4)

I) THE PRINCIPLE OF RELATIVITY (≡P.R.)

"All laws of physics are the
(i)

same in every inertial reference

frame"
(ii)

The scope ("in every inertial ... frame") and the strength of its declaration ("[applies to] all laws of physics") gives this principle universality and captures under its umbrella a diversity of implicit physical measurements.

Q: Where does the P.R. come from?

A: The P.R. is arrived at via a process of inductive reasoning applied to

(1.5)

(a) the observational evidence from comparisons between the outcomes of experiments performed in different inertial frames of reference.

(b) the relevant conceptual framework

(i) Laws of physics:

- Newton's 3 Laws of motion
- Maxwell's laws of electrodynamics
- Lorentz's law of motion for a charge in an e.m. field
- Thermodynamics
- etc

The observational evidence (1.6)
comes from comparing the results of experiments in different inertial frames of reference:

In two such frames consider two experiments with

1. Identical instructions
2. same experimental setup
3. same procedure
4. same data set
5. same data reduction

Then, within experimental errors (!), one will observe the same result.

From this particular pair of experiments, and others like it, one infers the following generalization: (1.7)

> In diffent inertial frames the same experiments yield, within experimental error, the same observed results.

This finding is the same regardless of whether the two experiments involve

- Newton's 3 Laws of motion
- Maxwell's laws of electrodynamics
- Lorentz's law of motion for a charge in an e.m. field
- Thermodynamics
- or any combination such laws

(1.8)

This finding, therefore, applies to all laws and one has

All laws of physics are the same in every inertial frame of reference

Thus

a) the form of these laws is the same in every inertial frame.

b) the numerical value of the physical constants

$(c, \hbar, e, m, k_{BOLTZ} \ldots)$

are the same w.r.t. every inertial frame

Restated negatively one has:

"The laws of nature do not provide a way of distinguishing one inertial frame from another"

Each of these statements is called the P.R.

II) INERTIAL FRAMES (1.9)

The relativity principle is a statement about the behaviour of things in different inertial frames.

Q: How can one tell such frames from non-inertial frames?

A: Within classical (i.e. non-quantum) mechanics the answer can be given by examining the measured sharp trajectories of free particles.

Consider the two reference frames depicted in Figure 1.2.

(1.10)

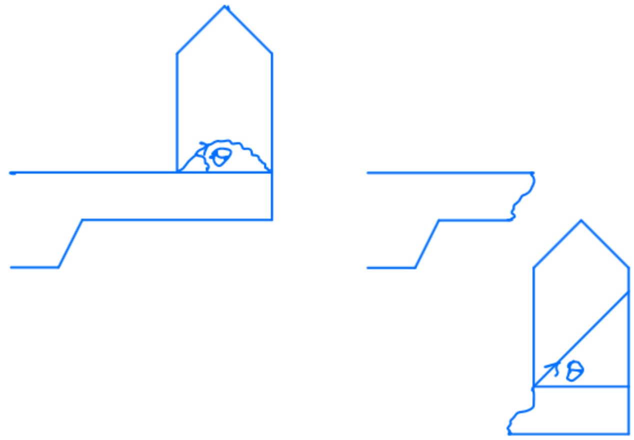

Figure 1.2: Two reference frames. A frame is said to be non-inertial whenever the motion of a free particle is observed to be non-uniform or not along a straight line.

An inertial frame is defined by Newton's 1st law of motion. (free particles remain at rest or in a state of uniform, straight-line motion)

1.) Definition (inertial, a.k.a. free float frame)

1.11

A frame is said to be *inertial* (or "*free float*") to the extent that all free particles in it comply with Newton's 1st law of motion.

2.) More explicitly, one has the following

Definition (*inertial frame*, a.k.a. *free float frame*)

Given: (i) a region of space and an interval of time

(ii) a set of freely floating particles in this region of spacetime

Figure 1.3: Particle motions in a region of spacetime.

(1.12)

Then: An inertial (= free float) frame is that region of spacetime coordinatized by a lattice work of clocks and measuring rods

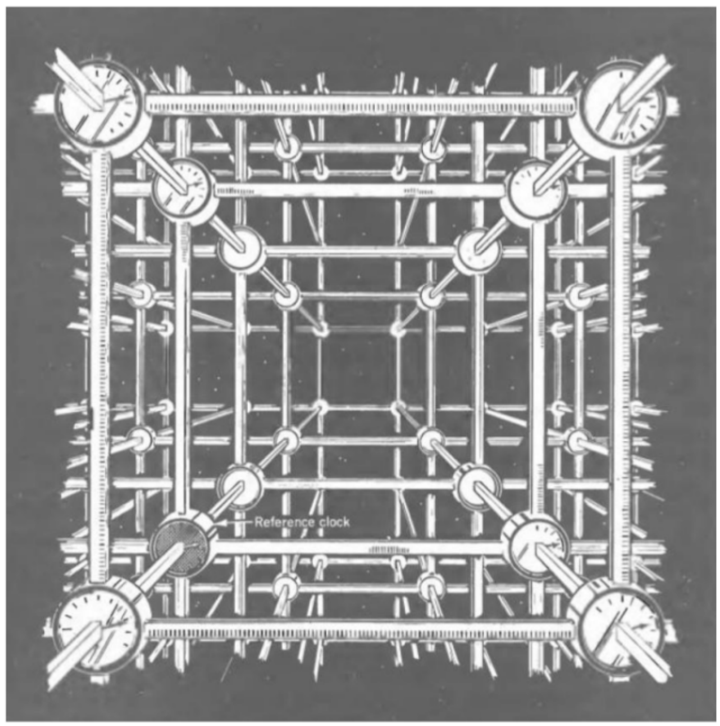

Figure 1.4

Lattice work of clocks and measuring rods

(1.13)

in such a way that - within some specified level of accuracy - the free particles travel

a) along straight lines

b) with constant velocity

for each and every particle in that region of spacetime

III) ISOTROPY OF SPACE

One of the surprising manifestation of the Principle of Relativity is the isotropy of light propagation in inertial frames in relative motion.

To appreciate this manifestation compare the propagation of sound with that of light in different inertial frames

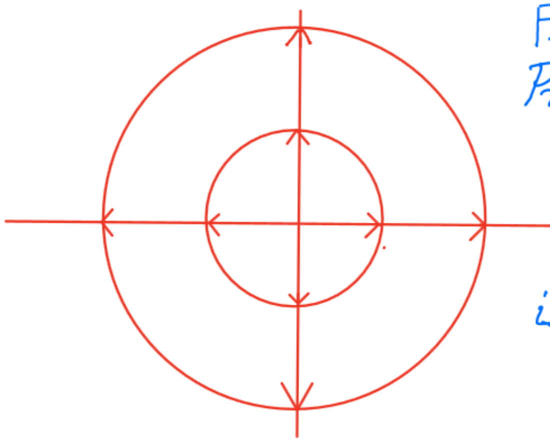

Figure 1.5(a) Propagating Phase fronts of <u>sound</u> in an isotropic medium

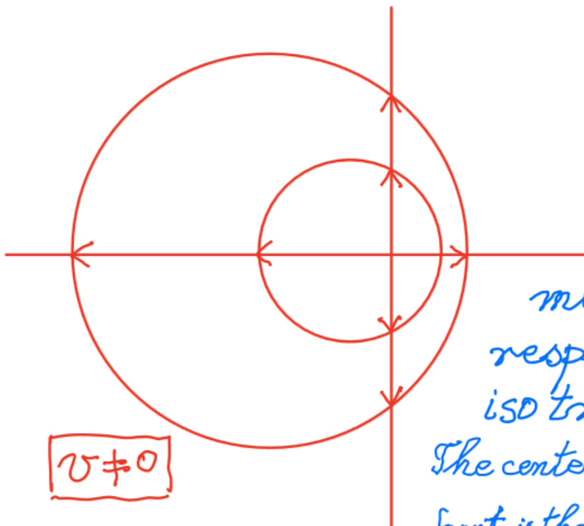

Figure 1.5 (b) Propagating Phase fronts of <u>sound</u> in a frame moving with respect to the above isotropic medium

The center of the large circular phase front is the location of the source in the medium at an earlier time (say at $t = -2$ sec.). By contrast, the center of the small circular phase is the the location of the source in the medium at an earlier time (say at $t = -1$ sec.).

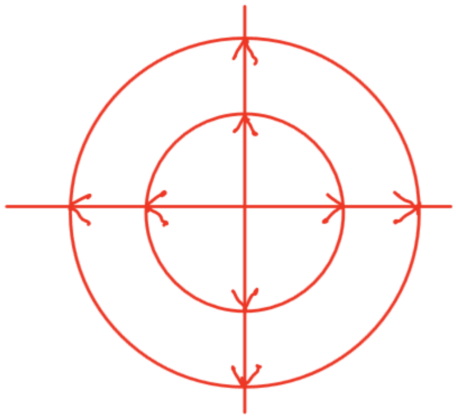

Figure 1.6 (a)

Propagating phase fronts of light in empty space

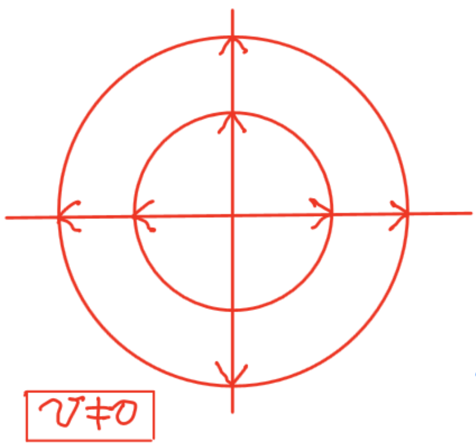

Figure 1.6 (b)

Propagating phase fronts of light in a frame moving w.r.t. the above frame

In a frame moving w.r.t. its medium ($v \neq 0$) sound waves propagate non-isotropically (different speeds in different directions) while e.m. waves still propagate isotropically. (same speed in all directions)

(1.16)

Thus one has the far-reaching result

Isotropy of space is frame independent

This principle is contained Maxwell's field equations. It also expresses the negative result of the Michelson Morley experiment

Isotropy of space says nothing about the <u>numerical value</u> of the speed of light. The Kennedy -Thorndike experiment says that also the magnitude of the velocity of light is frame independent.

1.17

Q: WHAT IS AN "OBSERVER"?

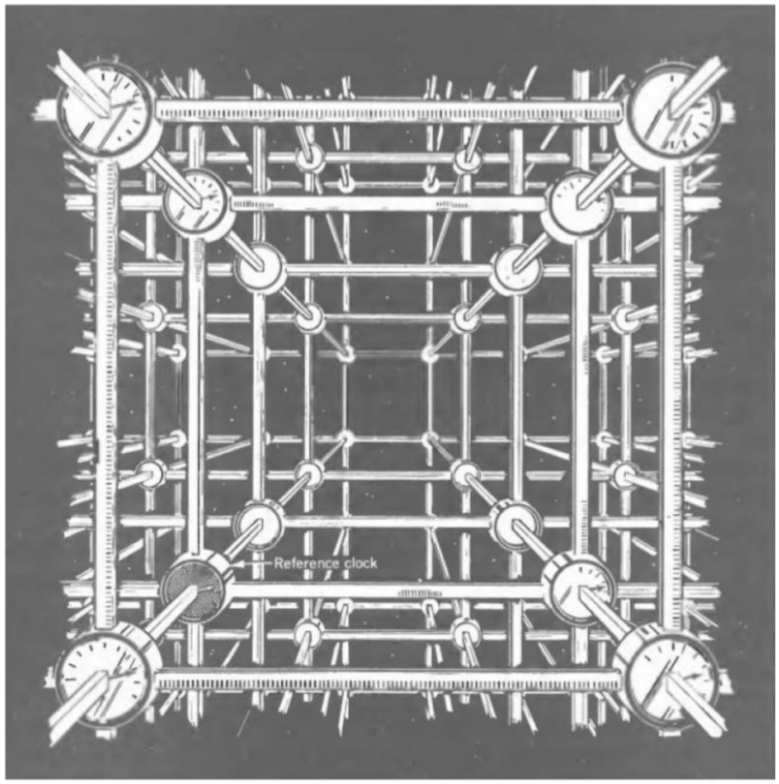

Figure 1.7: Latticework of clocks and measuring rods. It coordinatizes the local space time domain of an inertial reference frame as depicted above. Every such frame accommodates an agent (animate or inanimate entity) whose measurements are w.r.t. to such a lattice of clocks and rods. An *observer* refers to

A: (i) such an *agent* **together with** (ii) the *lattice-coordinatized frame* surrounding that agent. Thus an observer is an agent which resides in a local frame coordinatized by clocks and measuring rods.