LECTURE 4

(4.1)

T-W $\underbrace{6.1,\ 6.2,\ 6.3}_{2^{nd}\text{ editon}}$ $\underbrace{(1.7)}_{1^{st}\text{ edition}}$ of SPACETIME PHSICS

I. Lorentz spacetime & Lorentz transformation
II. Lorentz invariants
III. Lorentz metrology
IV. Region of spacetime: Its causal structure

I Lorentz Spacetime & Lorentz Transformation.

4.2

The geometrical correspondence between Euclidean space and Lorentzian spacetime is a guiding principle for expanding one's view of space and time into a unified whole, spacetime. Its comprising entities are events together with their pairwise relations: Timelike, Spacelike and Lightlike. In physics these events form arrays, lines, curves, surfaces, etc which reflect these relations. They are the same relative to all inertial frames, i.e. invariant upon transitioning between them.

Consider the Lorentz transformation which mathematizes such a transition

$$\begin{pmatrix}\bar t\\ \bar x\end{pmatrix} \rightsquigarrow \begin{pmatrix}t\\ x\end{pmatrix} = \Lambda \begin{pmatrix}\bar t\\ \bar x\end{pmatrix} = \begin{pmatrix}\bar t\, ch\theta + \bar x\, sh\theta\\ \bar t\, sh\theta + \bar x\, ch\theta\end{pmatrix}$$

and hence the spacetime loci of events

$$\boxed{\begin{aligned}&\bar x = 0 \Longleftrightarrow \frac{x}{t} = \frac{sh\theta}{ch\theta} \equiv \beta\\ \text{and}\quad &\bar t = 0 \Longleftrightarrow \frac{x}{t} = \frac{ch\theta}{sh\theta} = \frac{1}{\beta}\end{aligned}}\,.$$

These loci geometrize the old coordinate lines relative to the new:

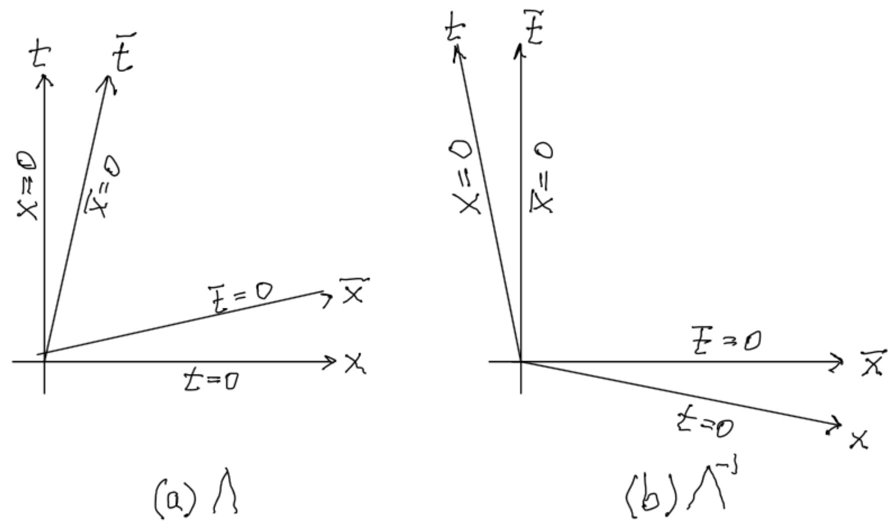


Figure 4.1: (a) Old coordinte lines $(\bar t, \bar x)$ relative to the new (t,x): Λ

(b) New coordinate lines (t,x) relative to the old $(\bar t, \bar x)$: Λ^{-1} [="inverse of Λ"]

4.3

II. Lorentz invariants

1.) Lorentz transformations are linear. This fact guarantees that parallelism (between straight worldlines, plane surfaces, etc) is an invariant.

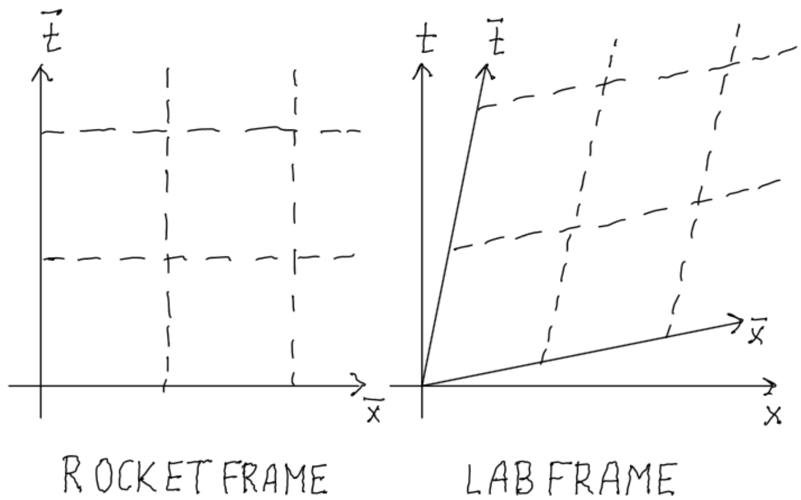


Figure 4.2: Invariance of parallelism under a Lorentz transformation

2.) Lorentz transformations preserve the invariance of the interval between a pair of events.

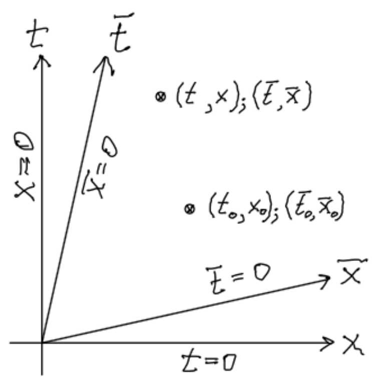


Figure 4.3: Two events coordinated in Lorentz frame S and $\bar{S}$.

4.4

Using the Lorentz transformation

$$t = \bar{t}\,ch\theta + \bar{x}\,sh\theta$$
$$x = \bar{t}\,sh\theta + \bar{x}\,ch\theta$$

the interval between these events is

$$(t-t_0)^2 - (x-x_0)^2 = (\bar{t}\,ch\theta + \bar{x}\,sh\theta - t_0)^2 - (\bar{t}\,sh\theta + \bar{x}\,ch\theta)^2$$

$$\vdots$$

$$\boxed{(t-t_0)^2 - (x-x_0)^2 = (\bar{t}-\bar{t}_0)^2 - (\bar{x}-\bar{x}_0)^2}$$

where one uses (do it as an instructive exercise!) the inverse transformation

$$t_0\,ch\theta - x_0\,sh\theta = \bar{t}_0$$
$$-t_0\,sh\theta + x_0\,ch\theta = \bar{x}_0$$

to obtain the last equality.

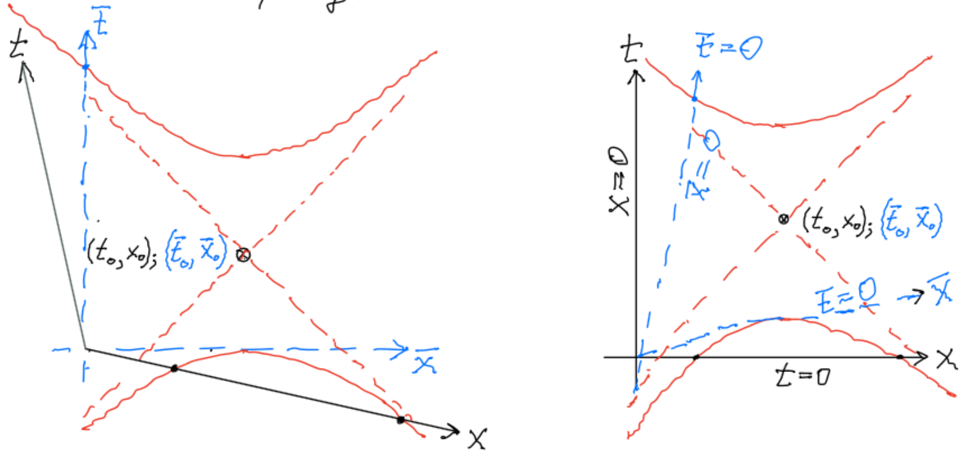


Figure 4.4: Invariant spacetime hyperbola centered around its spacetime center, the event $\{(t_0, x_0); (\bar{t}_0, \bar{x}_0)\}$. The dashed red lines make up the degenerate hyperbola. In four dimensional spacetime its upper and lower branch are the "future" and "past light cones". Their vertex is the center of the hyperbola.

4.5

A Lorentz invariant spacetime hyperbola corresponds to what in Euclidean space is a circle.

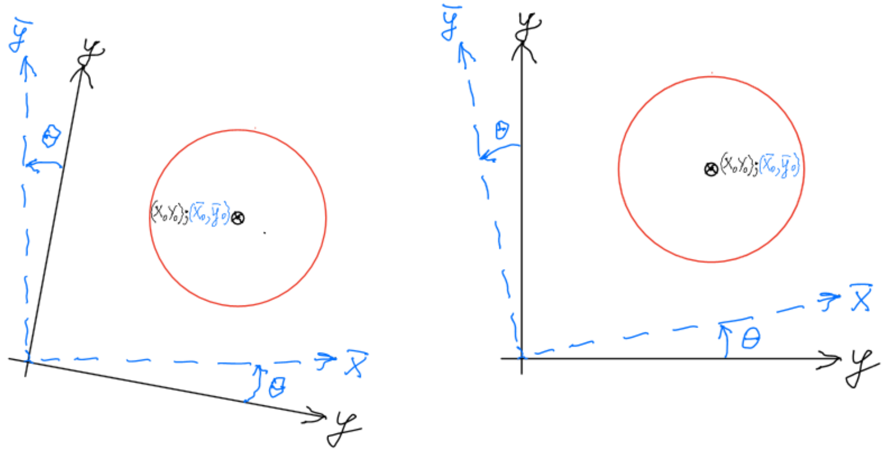


Figure 4.5: A Euclidean circle $(x-x_0)^2+(y-y_0)^2=(\bar{x}-\bar{x}_0)^2+(\bar{y}-\bar{y}_0)^2=r^2$, centered around the reference point $\{(x_0,y_0);(\bar{x}_0,\bar{y}_0)\}$, is invarient even though its coordinate representation has been changed by the Euclidean rotation.

III. Spacetime Metrology

The Lorentz invariant hyperbolas (= "circles" in spacetime) are the means to compare distances (= projections along some axis) in one frame with those in another frame. In particular

(i) compare two meter rods in their respective frames

(ii) compare two ticking clocks in their two respective frames.

The observed time dilation emerges as a result of the relativity of simultaneity. 4.6

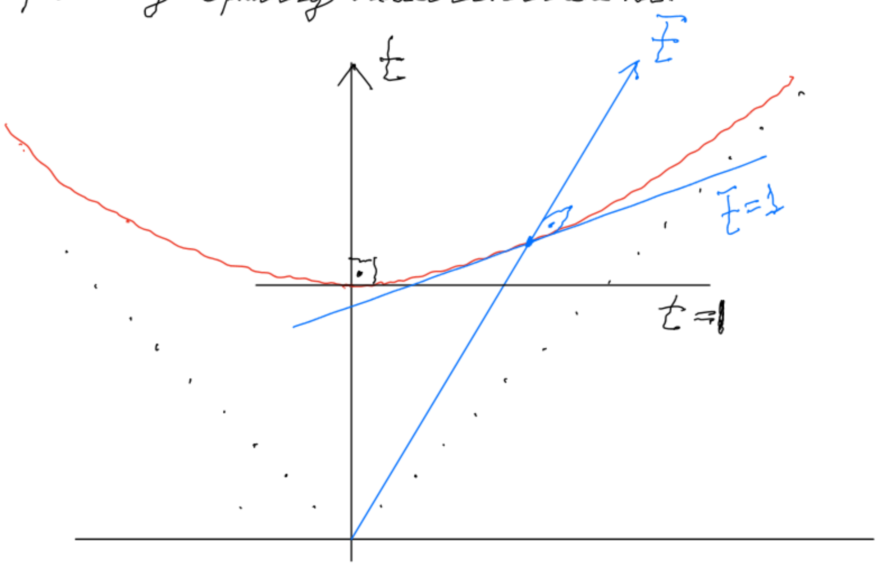
Example: Life time of a μ-meson

$\Delta\bar{t} = 2.2\times10^{-6}$ sec in its comoving ROCKET frame

$\Delta t = 333\times10^{-6}$ sec in the LAB frame on earth

$$\Delta t = \frac{\Delta\bar{t}}{\sqrt{1-\beta^2}} \Rightarrow \frac{1}{\sqrt{1-\beta^2}} = 157.9\,;\quad \beta = \operatorname{th}\theta = .999978;\ v = .999978\,c$$

Comparing the events simultaneous at $\boxed{\bar{t}=1}$ in the ROCKET frame with those simultaneous at $\boxed{t=1}$ in the LAB frame leads to the following equally valid conclusions:



Rocket observer: "My clock reads $\bar{t}=1$ <u>before</u> LAB clock reads $t=1$."
"LAB clock is slow!"

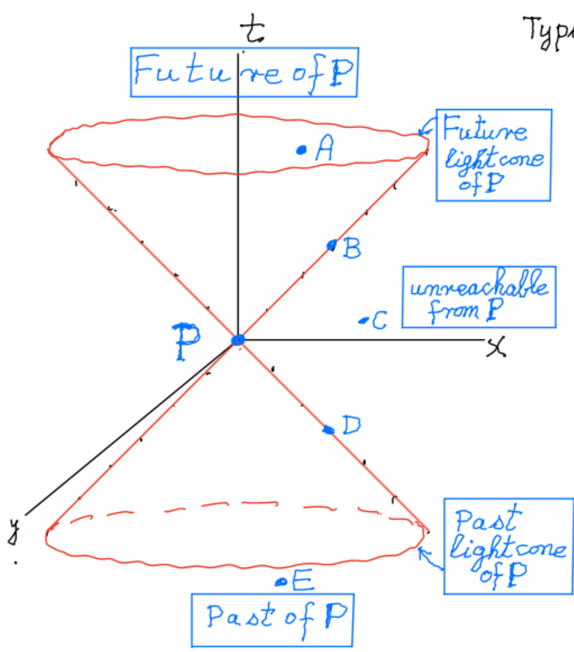
LAB oberver: "My clock reads $t=1$ <u>before</u> ROCKET reads $\bar{t}=1$.
"ROCKET clock is slow!"

4.7

IV. Regions of Spacetime: Its Causal Structure.

There is one outstanding feature which Euclidean space does not have, namely a so-called causal structure, according to which the relation between pairs of events is either timelike, lightlike, or spacelike.

This causal structure is put into quantitative form ("mathematized") by the sign of the interval between a chosen reference event and every other event in its neighborhood.



Typical event	$t^2-x^2-y^2-z^2$	Nature of interval between typical event and P	t
A	>0	timelike	$t>0$
B	$=0$	lightlike	$t>0$
C	<0	spacelike	$t \gtreqless 0$
D	$=0$	lightlike	$t<0$
E	>0	timelike	$t<0$