

Supplement to Lecture 11

SOME THOUGHTS ON THE WAVE MECHANICAL
SIGNIFICANCE OF ACCELERATED FRAMES.

9.7.a

3. The geometrical parallelism between
the Euclidean plane and the space
time of a pair of accelerated coordinate
frame ($\alpha \in \mathbb{S}$):

$$\begin{aligned} t &= \pm s \sinh \tau \quad +: (t, x) \in I \\ x &= \pm s \cosh \tau \quad -: (t, x) \in \text{II} \end{aligned}$$

~~II~~I

becomes obvious when one compares
the corresponding formulae for the
the distance and the interval between
points

$$ds^2 = r^2 d\theta^2 + dr^2 \quad \text{Euclidean plane}$$

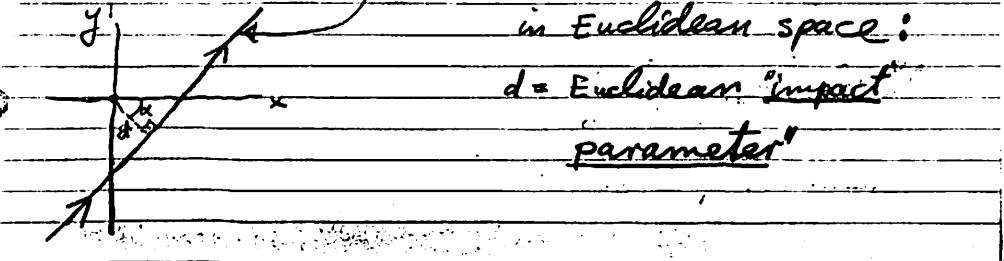
$$dr^2 = -s^2 d\tau^2 + ds^2 \quad \text{Lorentz plane}$$

Let us extend this comparison from
classical trajectories, where one has

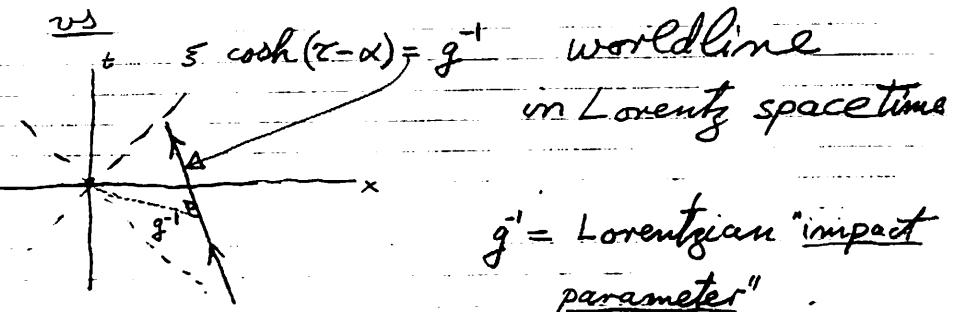
$$r \cos(\theta - \alpha) = d \quad \text{straight line}$$

in Euclidean space:

$d = \text{Euclidean "impact parameter"}$



9.7.b



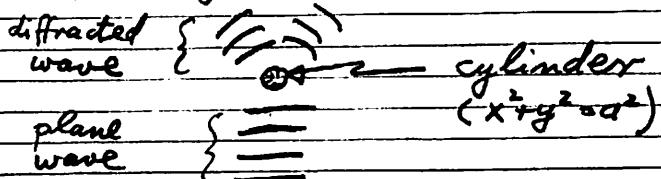
to the comparison of wave propagation
where one has

$$(\nabla^2 + k^2)\psi = \left(\frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} + k^2 \right) \psi = 0$$

for the Helmholtz equation in Euclidean space

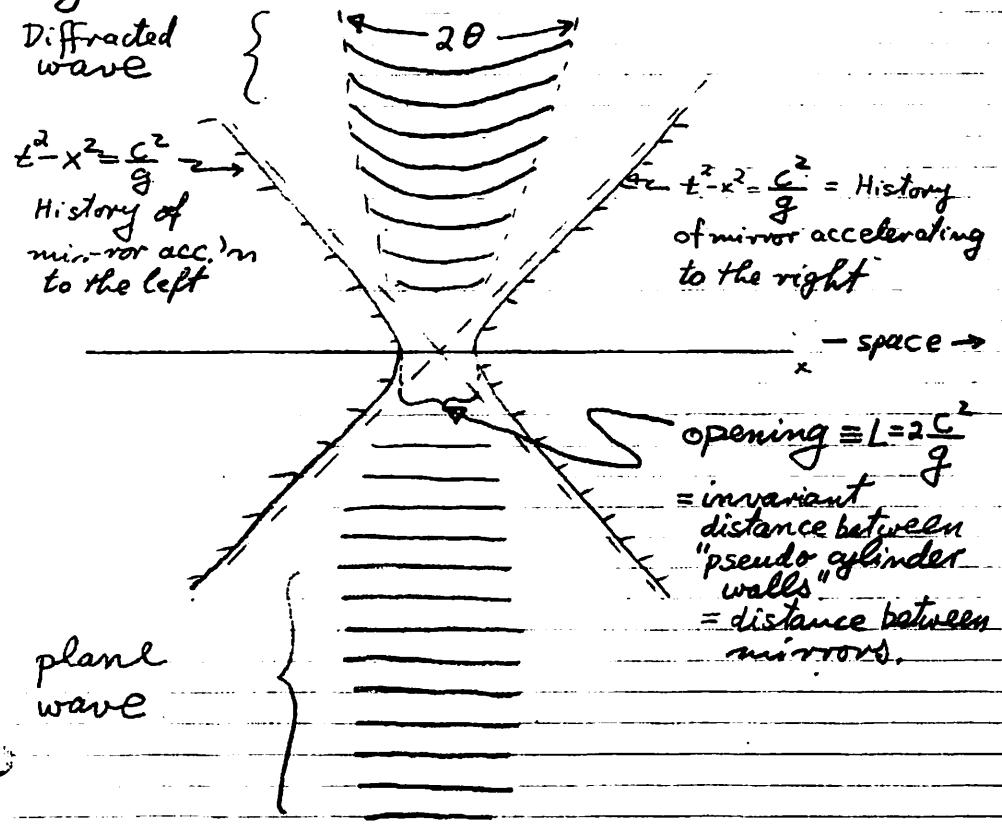
$$\boxed{\square - \left(\frac{mc}{r} \right)^2} \psi = \left[\frac{1}{s} \frac{\partial}{\partial s} \xi \frac{\partial}{\partial \xi} - \frac{1}{s^2} \frac{\partial^2}{\partial \xi^2} - k_c^2 \right] \psi = 0, \quad \text{with } k_c = \frac{mc}{r}.$$

for the scalar wave equation in Lorentz spacetime.
The comparison suggests that one consider
the scattering (i.e. "diffraction") of a plane wave in Euclidean
space off a cylinder



9.7.c.

and compare this scattering with the diffraction of a plane wave in Lorentz spacetime through the opening between the future and the past provided by a pair of accelerated mirrors



9.7.d.

We consider a plane wave propagating from the distant past towards the event at the origin which lies halfway between the accelerating mirrors. The opening at $t=0$ provided by these mirrors has a Lorentz invariant diameter given by

$$L = 2 \frac{c^2}{g}$$

This is the separation between their two histories

$$t = \pm g^{-1} \sinh \xi$$

$$x = \pm g^{-1} \cosh \xi$$

Upon emerging from between the two mirrors the wave has become a diverging beam in spacetime, or, equivalently, an expanding wave packet in space.

Its expansion rate is

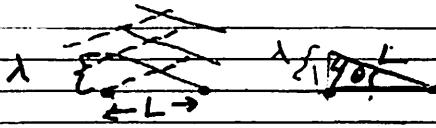
$$\frac{v}{c}, \quad \text{with } v = \pm g$$

9.7.e

which for small velocities one can equate to the angle of divergence of the beam in spacetime,

$$\frac{v}{c} \approx \theta$$

The divergence of the beam is due to the wave nature of beam emerging from the opening then propagating into the future. The fact that, except for a minus sign, the governing scalar wave equation is the same as the Helmholtz wave equation in Euclidean space, implies that one can apply the usual wave optics estimate to determine the divergence angle of the emerging beam.



$$\theta \propto \frac{\lambda}{L}$$

9.7.f

The wave length λ for a wave beam whose width expands non-relativistically ($\theta \approx \frac{v}{c} \ll 1$) is the Compton wavelength of the scalar wave equation,

$$\lambda = \frac{h}{mc}$$

The energy, more precisely, the fourth component of the energy-momentum four vector of a single quantum associated with the wave passing through the opening, is mc^2 .

The diffraction causes this particle to acquire a spatial component in its energy. This spatial component could be measured in the accelerated if the mirror were attached to a spring so that the mirror-spring

9.7.g

system would act as a particle detector. The recorded spatial component of the recoil energy would be

$$E \approx \frac{v}{c} mc^2$$

Combining the five boxed equations, one obtains

$$E \approx \frac{\Delta}{L} mc^2$$

$$E \approx \frac{(h/mc)}{2(c^2/g)} mc^2$$

$$E \approx \frac{h}{c} \frac{g}{2}$$

The remarkable aspect of this qualitative result is that this experimentally verifiable energy-acceleration relation is independent of the particle species and depends only on two constants of Nature, the quantum of action h and the speed of light c .

9.7.h

It seems clear that the framework of the discussion leading to this energy-acceleration relation calls for what in Euclidean space corresponds to (i) the scattering of particles and of waves by a cylinder, (ii) scattering cross section and (iii) differential cross section.

Such an investigation, which would be based on uniformly accelerated reference frames, is liable to uncover properties of matter (and of gravitation) which would have stayed hidden relative to inertial frames of reference.

It is possible to cite one astonishing example of this.

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4. A uniformly accelerated thermometer registers a non-zero value given by the Davies-Urrutia temperature

$$kT = \frac{\hbar}{c} \frac{1}{2\pi} g$$

in Minkowski spacetime, and by the Hawking temperature

$$kT = \frac{\hbar}{c} \frac{1}{2\pi} \left(\frac{G^4}{4\pi GM} \right)$$

in the spacetime of a black hole with mass M .

Here G is the universal gravitational constant, c the speed of light, \hbar the quantum of action, and k is Boltzmann's constant.

Why Nature presents us with such a temperature, and what its physical significance is, is still a mystery.