Lecture 32 Supplement

The meaning of torsion in the mechanics of elastic media:

dislocation density
What's the meaning of Torsion ≠ 0?
In elasticity theory one compares the strain configuration of a continuous medium before and after the application of some stress. At each point of the medium one introduces a set of basis vectors. In the unstrained configuration, the basis at one point is usually taken to be parallel to the basis at another point.

In a strained configuration, the basis vectors are no longer parallel in general.

Let $\mathbf{v}$ and $\mathbf{w}$ are vectors associated with point defects of a crystalline (or polycrystalline) medium.

\begin{align*}
\mathbf{v}, \mathbf{w} : \nabla_{\mathbf{v}} \mathbf{v}, \nabla_{\mathbf{w}} \mathbf{w} & = 0 \\
\text{i.e. } \Gamma^{r}_{kj} & = 0
\end{align*}

\textit{Strain induced covariant derivative.}

If the vector ("Burger's vector") obtained from the strain tensor $\nabla_{\mathbf{v}} \mathbf{v} - \nabla_{\mathbf{v}} \mathbf{v} - [\mathbf{v}, \mathbf{w}]$

associated with the strained state vanishes then one says that dislocations or rather the dislocation density is absent.

If the Burger vector

\[ T(\mathbf{v}, \mathbf{w}) = 0 \]

then this fact expresses the existence of a non-zero dislocation density.
If $T(W,V)$ points out of the plane spanned by $V$ and $W$, then one has a density of screw dislocations.

If $T(W,V)$ lies in the plane of $V$ and $W$, then one has a density of edge dislocations.

$$T(W,V) = 0$$

A single "edge dislocation."

For applications to Continuum mechanics, see, e.g., "Continuum Mechanics," by W. Jaureguy, or Theory of Elasticity, by Landau & Lifschitz.