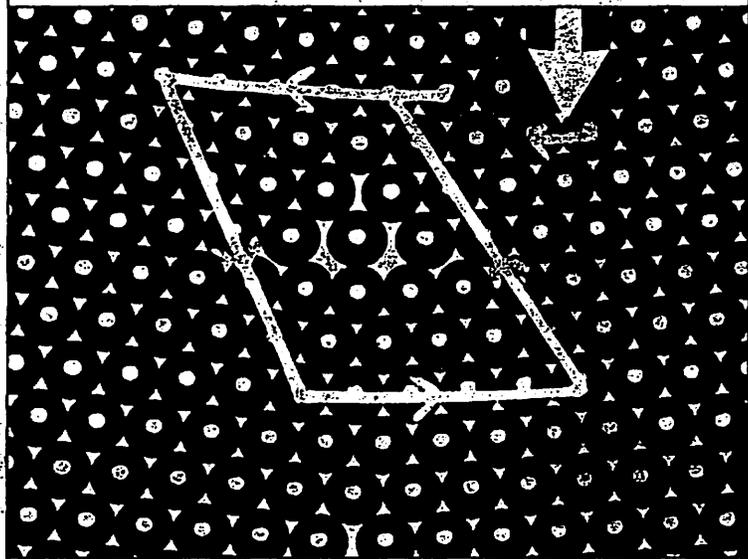


Lecture 32 Supplement

The meaning of torsion
in the mechanics of
elastic media:
dislocation density

What's the meaning of

Torsion $\neq 0$



2

Torsion = Dislocation Density ~~43-1~~
325.1

In elasticity theory one compares the strain configuration of a continuous medium before and after the application of some stress. At each point of the medium one introduces a set of basis vectors

In the unstrained configuration the basis at one point is usually taken to be parallel to the basis at another point.

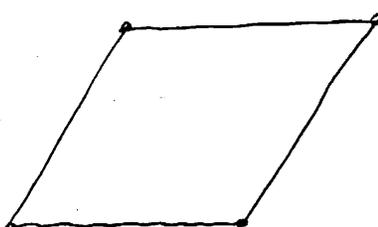
In a strained configuration

the basis vectors are however no longer parallel in general.

Let u and v are vectors associated with pairs of atoms of a crystalline (or polycrystalline) medium

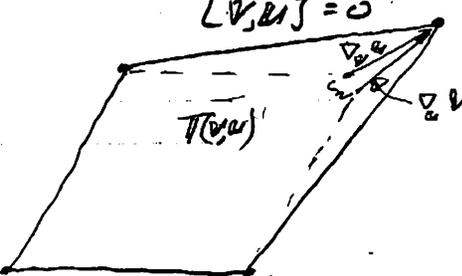
325.2
43-2

$[V, u] = 0$



unstrained state:
 $u, v : \nabla_u v, \nabla_v u = 0$
 i.e. $\Gamma^i_{jk} = 0$

$[V, u] = 0$



strained state
 $\nabla_u v, \nabla_v u \neq 0$
 i.e. $\Gamma^i_{jk} \neq 0$

STRAIN INDUCED COVARIANT DERIVATIVE

If the vector ("Burger's vector") obtained from the torsion tensor T ,

$$T(v, u) = \nabla_v u - \nabla_u v - [v, u],$$

associated with the strained state vanishes then one says that dislocations, or rather the dislocation density is absent.

If the Burger vector

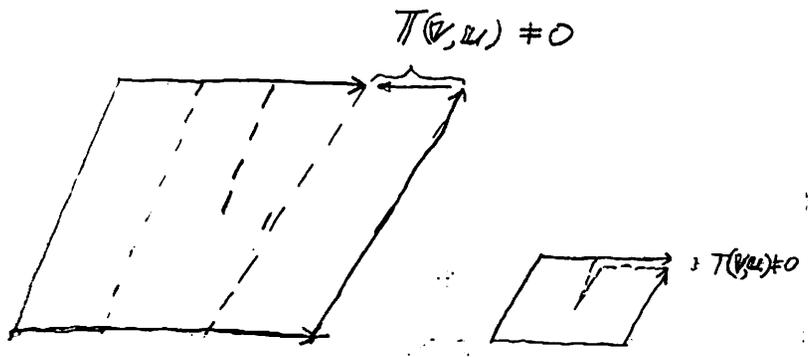
$$T(v, u) \neq 0$$

then this fact expresses the existence of a non-zero dislocation density.

SKIP
43-3
325.3

If $T(u, v)$ points out of the plane spanned by v and u then one has a density of screw dislocations

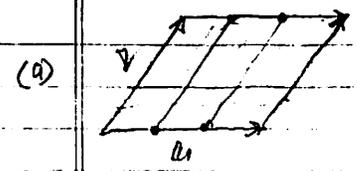
If $T(u, v)$ lies in the plane of v and u then one has a density of edge dislocations



A single "edge dislocation."

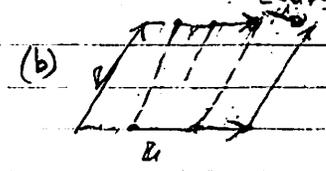
For applications to continuum mechanics see, e.g. "Continuum Mechanics", by W. Jaumann (QA808, 2 J3) or Theory of Elasticity, by Landau & Lifschitz

Let $T(u, v) = \nabla_u v - \nabla_v u - [u, v]$
consider strain induced parallel shift.



$\nabla_u v = 0$
 $\nabla_v u = 0$
 $[u, v] = 0$

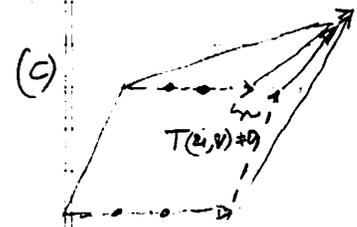
(Density of) dislocation absent



$\nabla_u v = 0$
 $\nabla_v u = 0$
 $[u, v] \neq 0$

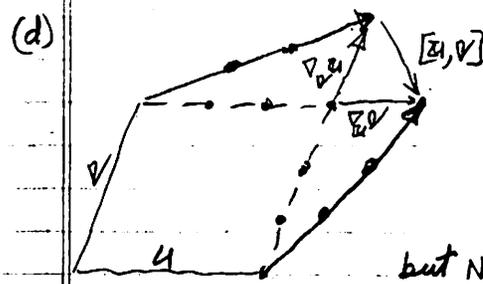
(Density of) dislocations
(a) screw dislocations
(b) edge dislocation

⇒ The Burger vector $T(u, v) \neq 0$



$\nabla_u v \neq 0$
 $\nabla_v u \neq 0$
 $[u, v] = 0$

Dislocations are present because $T(u, v) \neq 0$



$0 = \nabla_u v - \nabla_v u - [u, v] = T(u, v)$

$\nabla_u v \neq 0$
 $\nabla_v u \neq 0$
 $[u, v] \neq 0$

but No dislocation

$T(u, v)$ is the TORSION TENSOR (or Burger's vector) = amount of dislocation enclosed

By eq 1