

LECTURE 11

I) World Line vs World Tube

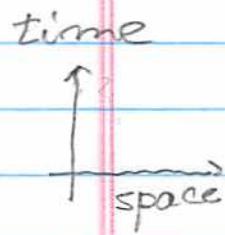
II) Density-flux of Charge and
Momenenergy and their Conservation

III) Physical meaning of the
Momenenergy Tensor components

IV Momenenergy Tensor for a
Mixture of Particles

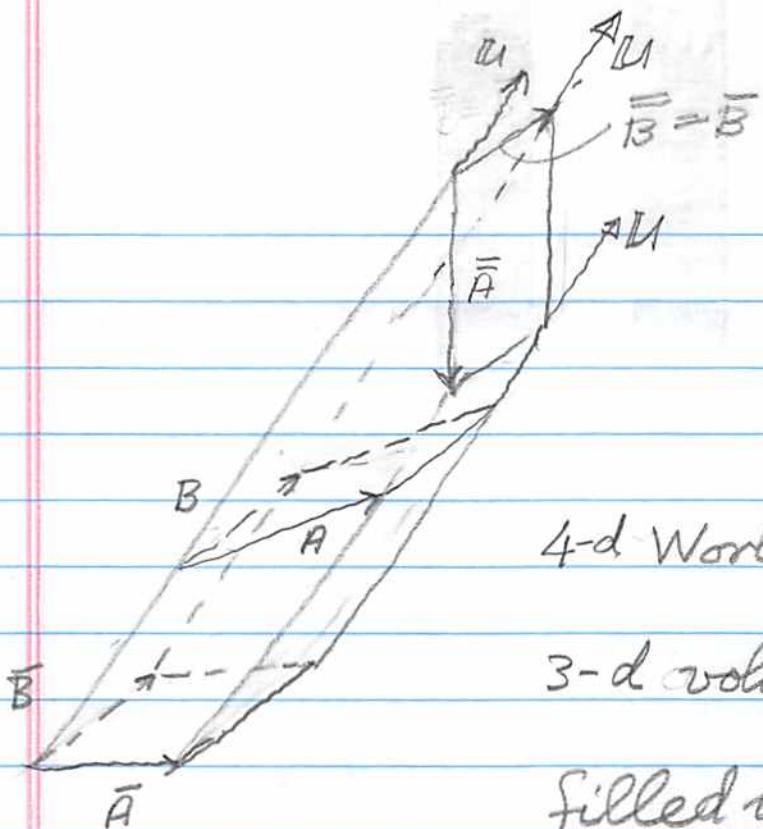
I) WORLD LINE vs WORLD TUBE -11.1-

Matter in motion consists of particles in motion. The existence of a single particle having 4-velocity $u^i = u^v e_v$ is mathematized in 4-d spacetime by a worldline whose tangent is u^i



Worldline of a
single particle

The existence of a set of particles aggregated into a continuous but finite cloud of matter moving with 4-velocity $u^i = u^v e_v$ is mathematized in 4-d spacetime by a worldtube whose tangent is u^i .



4-d Worldtube having
3-d volume cross sections
filled with a cloud
of particles having 4-velocity

u^μ .

Comment

○ Conservation of Particles Mathematized,

If particles are neither destroyed nor created

(e.g. worldline don't terminate nor spring into existence),

then

$${}^*S(\bar{A}, \bar{B}, \bar{C}) = {}^*S(A, B, C) = {}^*S(\bar{A}, \bar{B}, \bar{C})$$

Here
$${}^*S = \underbrace{N u^\nu}_{\text{observed}} \epsilon_{\nu\alpha\beta\gamma} \frac{dx^\alpha dx^\beta dx^\gamma}{3!} = S^\nu \Sigma_\nu \quad (11.1)$$

is the particle density-flux of the moving cloud.

II) DENSITY-FLUX of CHARGE and MOMENERGY

Each particle in a moving aggregate has

intrinsic properties (attributes) such as its restmass,

$$m \left[\frac{\text{amount of restmass}}{\text{particle}} \right]$$

its charge,

$$q \left[\frac{\text{amount of charge}}{\text{particle}} \right]$$

its momenergy,

$$p = m u \left[\frac{\text{amount of momenergy}}{\text{particle}} \right]$$

and other attributes, such as intrinsic

angular momentum ("spin") and

others.

In the context of continuous matter in motion the existence of these attributes imply that a given particle density-flux

^{*5, Eq.(11.1)}

is matched by a corresponding density-flux

of the attribute carried by the particles comprising this continuous matter:

$$(i) * \mathbb{J} = q^* S \quad \text{"charge density-flux 3-form"}$$

$$= q N u^\nu \epsilon_{\alpha\beta\gamma} dx^\alpha \wedge dx^\beta \wedge dx^\gamma$$

$$= \mathbb{J}^\nu \sum_m$$

with

$$\mathbb{J} = p \otimes \mathbb{T}$$

$$\mathbb{J} = J^\nu e_\nu \text{ and } J^\nu = q N u^\nu$$

as the corresponding charge current

4-vector and its components, and

$$(ii) * T = p \otimes S \quad \text{"momentum density-flux 3-form"}$$

$$= e_\mu p^\mu N u^\nu \epsilon_{\nu\alpha\beta} dx^\alpha \wedge dx^\beta \wedge dx^\gamma$$

$$= e_\mu T^{\mu\nu} \otimes \sum_m$$

with

$$T = e_\mu \otimes T^{\mu\nu} e_\nu \text{ and } T^{\mu\nu} = p^\mu S^\nu$$

$$= u^\mu m N u^\nu$$

as the corresponding momenenergy
Tensor and its components.

The difference between these density-flux
 3-forms is that

(i) ${}^*S, {}^*J$ are tensors of rank $(\frac{0}{3})$, in particular,
 they are scalar-valued ("particle,"
 charge") 3-forms and
 (the corresponding)
 S, J are vectors, tensors of rank $(\frac{1}{3})$.

(ii) *T is a tensor of rank $(\frac{1}{3})$, in particular,
 it is a vector-valued ("momenenergy")
 3-form and

T is the corresponding tensor of rank $(\frac{2}{3})$.

Comment

Charge and Momentum Conservation

for a Cloud of Uniformly Moving Particles

(i) If charge is neither destroyed nor created, then

$${}^*J(\bar{A}, \bar{B}, \bar{C}) = {}^*J(A, B, C) = {}^*J(\bar{A}, \bar{B}, \bar{C})$$

(ii) If momentum is neither destroyed nor created, then

$${}^*T(\bar{A}, \bar{B}, \bar{C}) = {}^*T(A, B, C) = {}^*T(\bar{A}, \bar{B}, \bar{C}).$$

III) PHYSICAL MEANING OF THE

MOMENERGY TENSOR COMPONENTS

$$T^{\mu\nu} = p^\mu s^\nu$$

s^0 = observed particle density

s^k = observed flux of particles
into the k^{th} direction

$$= \frac{\text{particles}}{(\text{time})(\text{area with normal into } k^{\text{th}} \text{ dir.})}$$

T^{00} = observed energy density

$$= \frac{\text{energy}}{\text{volume}}$$

T^{0k} = (energy flux into k^{th} direction)

$$= \frac{(\text{energy})}{(\text{time})(\text{area into } k^{\text{th}} \text{ direction})}$$

$$= \frac{(\text{power})}{(\text{area into } k^{\text{th}} \text{ direction})}$$

T^{i0} = observed density of i^{th} momentum component
= (momentum into i^{th} direction)
(volume)

T^{ik} = flux of i^{th} mom. component into k^{th} direction
= (i^{th} mom. component)
(time) (area into k^{th} direction)

Summary

S^0, T^{0M} refer to the density of "stuff".

S^k, T^{kM} refer to the flux of "stuff",
"stuff"
(time)(area)

IV MOM ENERGY TENSOR for a MIXTURE OF DIFFERENT PARTICLE,

Consider the circumstance where the
3-d spacetime volume spanned by (A, B, C)
contains several species of particles,
each having respective 4-velocities

u_1, u_2, \dots

momenergies

p_1, p_2, \dots

invariant densities

N_1, N_2, \dots

and rest masses

m_1, m_2, \dots

Then the total amount of momenergy piercing the spacetime

A, B, C is

$${}^*T(A, B, C) = \sum_A P_A \otimes {}^*S_A(A, B, C)$$

$$= e_\mu \left(\underbrace{\sum_A T^{^\mu\nu}_A}_{T^{\mu\nu} \text{ TOTAL}} \right) d^3 \Sigma_\nu(A, B, C)$$

Here

$$\sum_A T^{^\mu\nu}_A = \sum_A P_A N_A u_A^\mu u_A^\nu$$

$$= \sum_A m_A N_A u_A^\mu u_A^\nu$$

$$= T^{\mu\nu}$$

are the components of the total

momenergy tensor $T^{\mu\nu}$

$$T = e_\mu \left(\underbrace{\sum_A m_A N_A u_A^\mu u_A^\nu}_{T^{\mu\nu}} \right) e_\nu$$
$$= e_\mu \otimes e_\nu T^{\mu\nu}$$

while

$$N_A U_A^\vee \sum_r (A, B, C) =$$

$$= N_A U_A^\vee \text{E}_{\alpha\beta\gamma} dx_1^\alpha dx_2^\beta dx_3^\gamma (A, B, C)$$

is the number of type A particle world lines which pierce the positively oriented 3-volume (A, B, C) into the positive direction.

Summary

$${}^*T(A, B, C) = \sum_A P_A \otimes {}^*S(A, B, C)$$

= amount of momenergy piercing in volume spanned by (A, B, C)

$${}^*T = \sum_A P_A \otimes {}^*S = \left[\frac{\text{(momenergy)}}{\text{(as-yet-unspecified spacetime 3-volume)}} \right]$$

$$T = \sum_A P_A \otimes S \left[\begin{array}{l} \text{momenergy in an} \\ \text{as-yet-unspecified} \\ \text{oriented 3-volume} \end{array} \right]$$