

LECTURE 26

I Surface force density,

II Translational equilibrium

III Rotational kinematics

IV Lever arms.

V Moment of force.

For LECTURE 27

Reading assignment:

1. Type set LECTURE 24 'hand'

2. MTW §15,3

I.) SURFACE FORCE DENSITY 26.1

From the mechanics of a rigid body subjected to force fields one knows that they have two causal attributes:

- (1) those that result in translational motion and
- (2) those that result in rotational motion of a given body.

To mathematize the difference and the relation between the two, concretize the force field by means of an electrostatic field interacting with a dielectric.

A dielectric consists consists of an array of polarizable molecules, each having a dipole moment

$$\vec{d}(\vec{E}) = \vec{e}_m \gamma^m(\vec{E}) q$$

The driving force behind deploying mathematizing these concepts is that their extension to 4-d spacetime is what is needed in order to understand the $\vec{E}, \vec{H}, \vec{E}_1$, in particular the L, h, s, \dots , i.e. the Einstein tensor

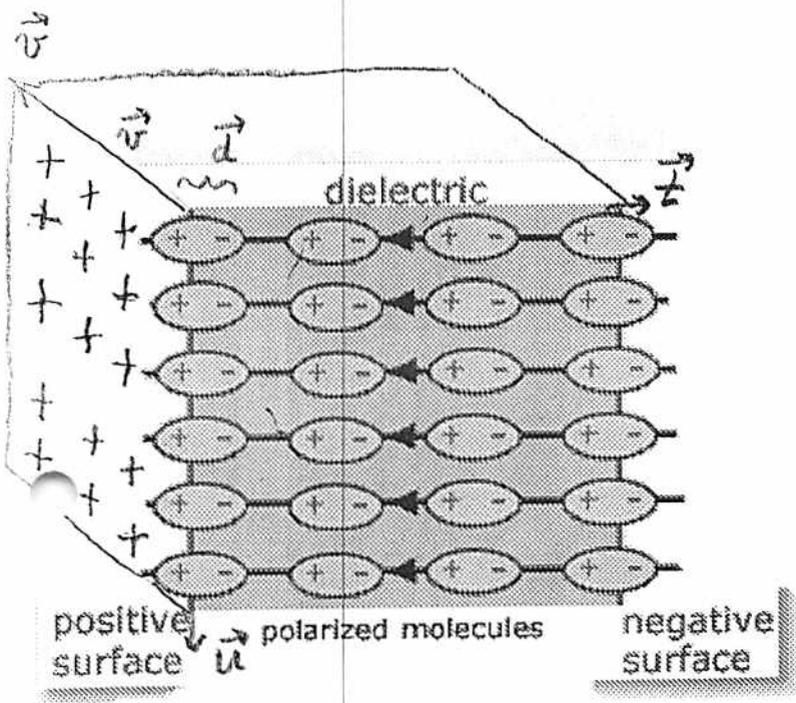


Figure 26.1

Polarized dielectric cube in an electric field

$$\vec{E} = \epsilon_r \vec{E}_R$$

when subjected to a homogeneous electrostatic field

$$\vec{E} = \vec{E}_0 E^k$$

For a dielectric cube having volume

$$\epsilon_{ijk} dx^i \wedge dx^j \wedge dx^k (\vec{u}, \vec{v}, \vec{t})$$

spanned by the triad of vectors $(\vec{u}, \vec{v}, \vec{t})$,

the force acting on the (\vec{u}, \vec{v}) -spanned

face is

of molecules
on a (\vec{u}, \vec{v}) -face

$$\text{"force"} = \vec{F}(\vec{u}, \vec{v}) = \vec{E} q N \underbrace{\epsilon_{mlij} dx^i \wedge dx^j}_{\text{surface density}} (\vec{u}, \vec{v})$$

$$\underbrace{\vec{F}_{ij}}_{\text{surface density}} dx^i \wedge dx^j$$

Thus one has the concept

$$\vec{F}_{ij} \equiv \frac{\vec{F}_{ij}}{2!} dx^i \wedge dx^j = \frac{\text{(force)}}{\text{(area)}} = \text{"surface" force density}$$

This is a stress field. It acts on all faces of the rigid cube. It has two causal attributes which

1. result in the cube's translational motion, and
2. result in the cube's rotational motion.

II) TRANSLATIONAL EQUILIBRIUM

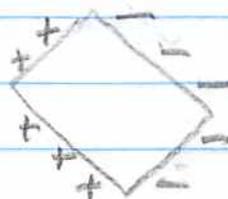
A dielectric cube with zero total

charge will experience a zero total

force from the (homogeneous) stress

field of a (homogeneous) electric field

acting on the cube



$$\vec{F} = Q_p \vec{E}^k$$

The sum total of the forces acting on the six faces vanishes

$$(26.1) \quad \vec{F}_{\text{tot}} = \sum_{\ell=1}^6 \vec{F}_{\ell} (\ell^{\text{th}} \text{ face}) = 0$$

The boundary $\partial \mathcal{D}$ of the cube's interior domain \mathcal{D} consists of the union of its 6 faces

$$\partial \mathcal{D} = \bigcup_{\ell=1}^6 (\ell^{\text{th}} \text{ face})$$

and they come in pairs of opposing faces having opposite orientation;

Evaluating \vec{F}_{ℓ} on each pair

$$(u, v), (v, u); (v, t), (t, v); (t, u), (u, t)$$

one finds that

$$\sum_{\ell=1}^6 \vec{F}_{\ell} (\ell^{\text{th}} \text{ face}) = d \vec{F}_{\ell} (u, v, t)$$

The condition for translational equilibrium, Eq. (26.1), holds for all cubes spanned by triads of vectors such as $\{u, v, t\}$. Consequently, translational equilibrium is mathematized by

$$(26.2) \quad 0 = d\vec{F}_m = (F_{ij,k} + F_{jk,i} + F_{ki,j}) dx^i \wedge dx^j \wedge dx^k$$