

LECTURE 6

I. Electromagnetism vs gravitation;

Equations of motion and Field equations

II. Matter and Motion;
Its thinkers in history

III. Momentum:

1. Newtonian
2. Relativistic

Read Chapter 2 in 1st Edition

of SPACETIME PHYSICS
by Taylor & Wheeler

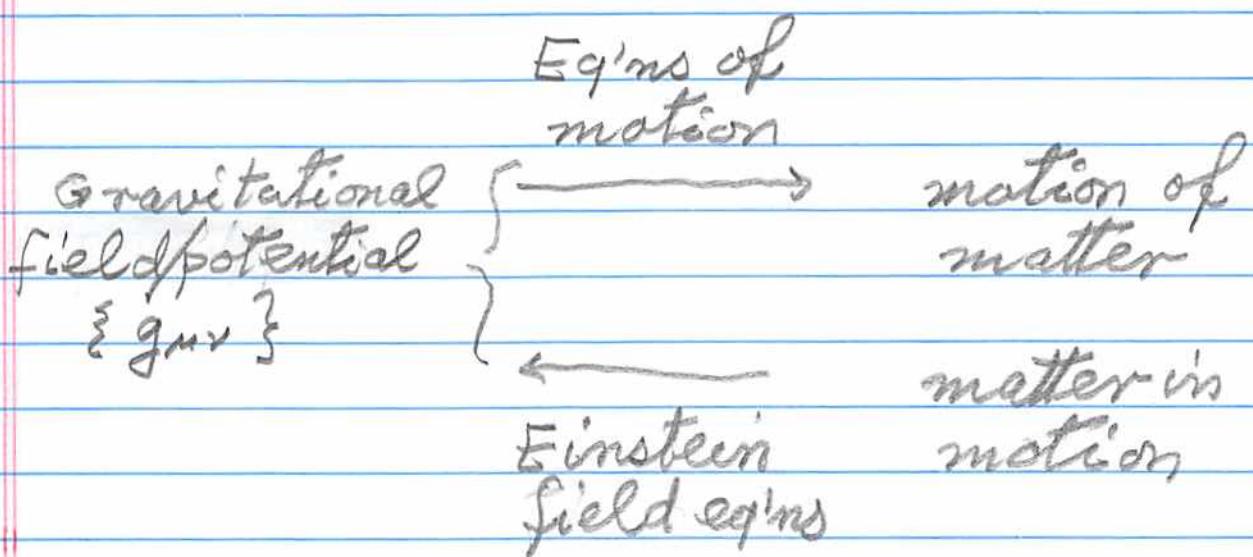
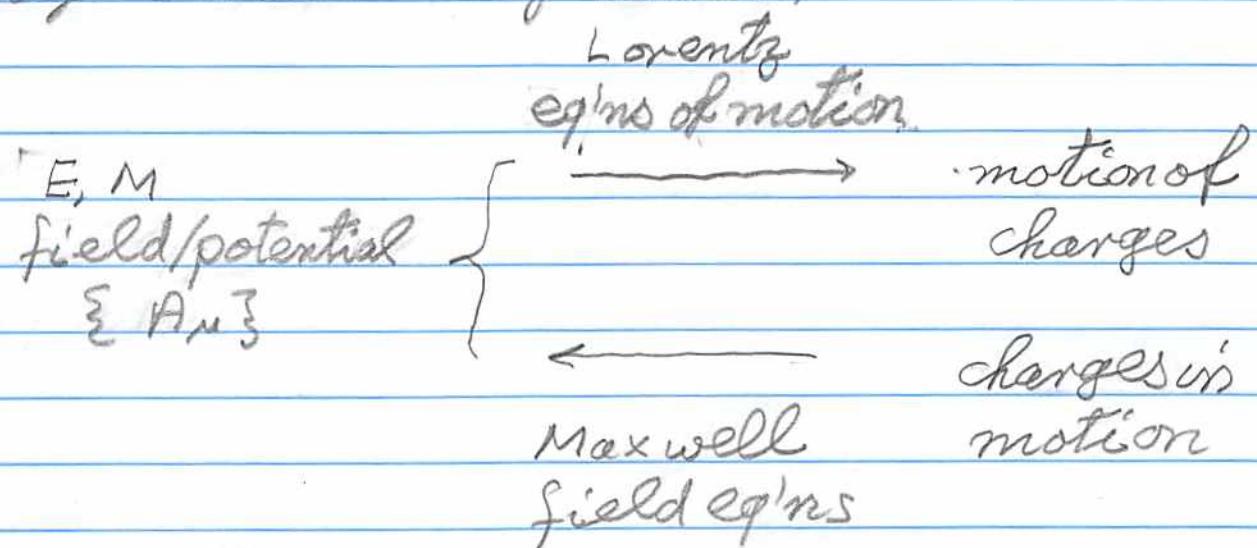
OR

Chapter 7 in 2nd Edition

of SPACETIME PHYSICS
by. T&W,

Chapter 6 in A JOURNEY INTO GRAVITY
AND SPACETIME by J.A. Wheeler

I) In regard to the relation between cause and effect the conceptual structure of the mathematization of gravitation is the same as that of electromagnetism



II. MATTER and MOTION.

6,2

In order to mathematize gravitation one must mathematize the motion of bodies, more generally of matter.

This is because gravitation leaves its perceptible imprint in the form of the motion of matter.

Subsequently we shall mathematize (by means of the Einstein field eq'n's) gravitation as a response to matter in motion.

The first thinkers to identify the concepts of matter and of motion were the Greek philosophers, even before Socrates, (Heraclitus, Parmenides, Zeno, Democritus,..)

and then most importantly Aristotle

He reconciled Parmenides's and Heraclitus's seemingly irreconcilable conclusions about things by means of the application of his law of identity (...): Things act in accordance with their nature.

These concepts were put into mathematical form by Galileo, Kepler and most definitively by Newton with his laws of dynamical motion.

Newtonian dynamics

1. Every object in a state of uniform motion tends to remain in that state unless a force is applied to it.

$$2. \quad \overrightarrow{\text{Force}} = \frac{d}{dt} (\overrightarrow{\text{momentum}})$$

3. For every action there is an equal and opposite reaction

Question: What facts of reality give rise to the concept momentum?

Answer: The nature of collision processes, as mathematized on page 6.6 using Newton's 3 laws of motion.

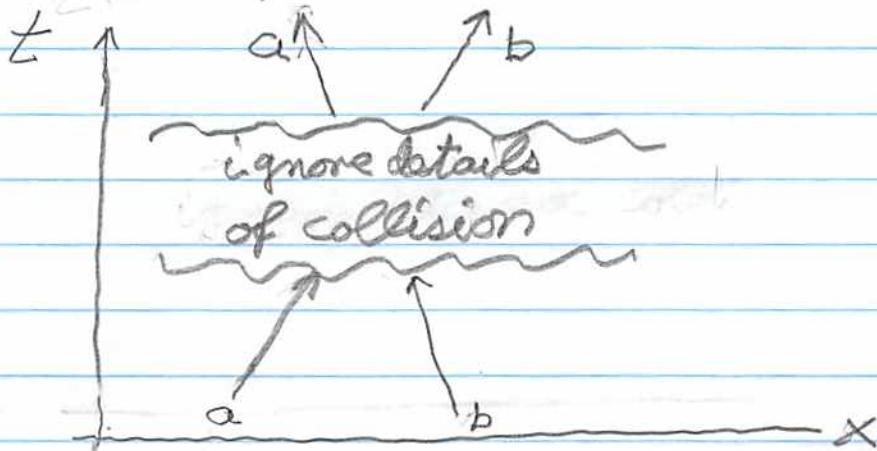
Using special Relativity, Einstein
not only generalized Newton's dynamical
laws of motion to relativistic velocities, but
also put these laws into geometrical
relativistic form by introducing the
concept of energy-momentum (also
known as momenergy) fourvector
into the dynamics of relativistically
moving matter.

III. MOMENTUM

1. One arrives at the definition of momentum for low velocity collision by means of the following Newtonian line of reasoning.

Q: What quantity common to all collision processes remains the same before and after any such process.

A: a) Consider the following spacetime process involving the collision of two particles



b) Consider the quantity

(mass) \times (velocity) \equiv Momentum

c) Newton's 2nd Law

$$\frac{d}{dt}(\text{momentum of } a) = F_{\text{bona}}$$

$$(\text{mom. of } a)_{\text{after}} - (\text{mom. of } a)_{\text{before}} = \int F_{\text{bona}} dt$$

$$d) F_{\text{bona}} = -F_{a \text{ on } b}$$

e) Apply c) & d) to all particles, and conclude

$$\sum_{\substack{\text{particles} \\ a, b}} (\text{momentum before}) = \sum_{\substack{\text{particles} \\ a, b}} (\text{momentum after})$$

2. Generalize the concept of momentum to relativistic processes:

A. Consider (mass)(velocity) and then label

$$m \frac{d \vec{v}}{dt} = \vec{p} \quad \text{momentum}$$

as "momentum".

However, this is uninteresting because this extension of Newton's definition does not

6.8

express any conserved quantity

before and after

Hence focus attention on those quantities

that do exhibit conservation.

B. Definition of relativistic momentum as validated by Tolman-Ehrenfest.

(FYI: A definition is the condensation of a vast body of observations - and stands or falls with the truth or falsehood of the observations.)

Theorem (Definition of momentum)

- Given:
- Principle of Relativity
 - Isotropy of space
 - Symmetry
 - \exists a unique momentum vector associated with a particle having a given velocity
 - the momentum is conserved during a collision
 - the correspondence at low velocities with Newtonian definition must not be violated.

Conclusion: $\overrightarrow{\text{momentum}} = m \frac{\vec{v}}{\sqrt{1-v^2}}$; \vec{v} =velocity

Comment about proof:

- Look at a very symmetric collision from the point of view of two reference frames
- Look for a quantity conserved in both frames to find the momentum-velocity relationship

proof: ① Q: what is the relation between \vec{P} , momentum, and \vec{v} , the velocity, of the particle?

$$A: \boxed{\vec{P} = f(v) \vec{v}}$$

Why? (i) isotropy of space



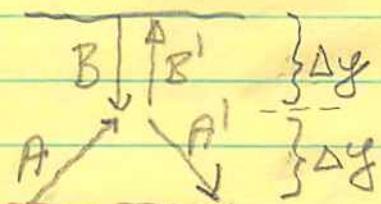
(ii) \vec{P} is unique

$$(iii) f(v) = f(\sqrt{v_x^2 + v_y^2 + v_z^2})$$

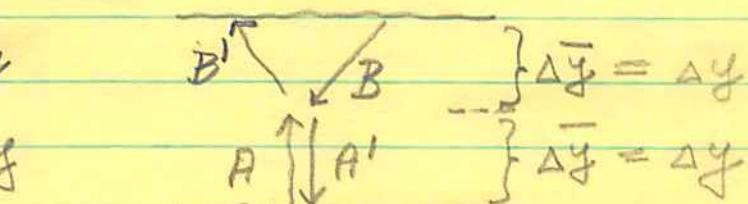
because of isotropy of space

② Principle of Relativity $\Rightarrow f(v)$ is the same fn in all inertial reference frames.

③ To determine f , consider the elastic collision between two identical particles viewed in two inertial frames



L.F.A.B



$\xrightarrow{\text{motion}}$

MOVING FRAME

For this type of a collision scenario one has

- ✓ particles are identical \Rightarrow the two
- ✓ Principle of Relativity \Rightarrow pictures are
- ✓ isotropy of space \Rightarrow symmetrical
- ✓ symmetry \Rightarrow pictures are congruent

(i) In the MOVING FRAME the components of A are

A's velocity = $\frac{\Delta \vec{r}}{\Delta t}$, where Δt = time for A to move from bottom to the pt of collision.

(ii) In the LAB FRAME the components of A are

A's four-vector = $(\Delta t, \Delta x, \Delta y, 0)$

$$= \left(\frac{\Delta \vec{r}}{\sqrt{1-\beta^2}}, \frac{\beta \Delta \vec{r}}{\sqrt{1-\beta^2}}, \Delta \vec{y}, 0 \right)$$

A's spatial velocity = $(\frac{\Delta x}{\Delta t}, \frac{\Delta y}{\Delta t}, 0)$

$$= (\beta, \frac{\Delta \vec{r}}{\Delta t} \sqrt{1-\beta^2}, 0)$$

$$= (\beta, \mu \sqrt{1-\beta^2}, 0)$$

where $\mu = \frac{\Delta \vec{y}}{\Delta t}$

② Conservation of momentum in LAB FRAME.

(2) along the X-direction in LAB FRAME:

$$\sum p_x)_{\text{before}} = \sum p_x)_{\text{after}}$$

$$f(\sqrt{\beta^2 + \mu^2(1-\beta^2)})\beta + 0 = f(\sqrt{\beta'^2 + \mu^2(1-\beta'^2)})\beta' + 0$$

A

B

A'

B'

No new information

(2) along the Y-direction in LAB FRAME

$$\sum p_y)_{\text{before}} = \sum p_y)_{\text{after}}$$

$$f(\sqrt{\beta^2 + \mu^2(1-\beta^2)})\mu\sqrt{1-\beta^2} - f(\sqrt{0+\mu^2})\mu =$$

A

B

$$f(\sqrt{\beta^2 + \mu^2(1-\beta^2)}) \leftrightarrow \mu\sqrt{1-\beta^2} + f(\sqrt{0+\mu^2})\mu$$

A'

B'

The resulting functional identity yields

$$f(\sqrt{\beta^2 + \mu^2(1-\beta^2)}) = \frac{f(\mu)}{\sqrt{1-\beta^2}}$$

Take the limit $\mu \rightarrow 0$ and obtain

$$\boxed{f(\beta) = \frac{f(0)}{\sqrt{1-\beta^2}}}$$

④ The value $f(0)$ is obtained from the Newtonian correspondence limit. Asymptotically one has

$$f(v) v_y \rightarrow m v_y$$

$$I = \frac{p_x}{p_y} = \lim_{v \rightarrow 0} \frac{m v_y}{f(v) v_y} = \frac{m}{f(0)}$$

Conclusion

$$\overrightarrow{\text{momentum}} = \frac{m}{\sqrt{1-\beta^2}} \vec{\beta}$$

Comment: If $f(v)$ is independent of v then one implicitly assumes the Newtonian (non-relativistic) approximation.