Lecture 24 Appendix

1. The eight bounding cubes of a 4-D domain

2. The trivector-valued moment of rotations on their 48 faces.
Consider an $n$-dimensional hypercube $S^2$ having volume $\sqrt{n}$ centered at the origin of the coordinate system, and having its 16 vertices located at:

\[\left(\pm\frac{\sqrt{n}}{2}, \pm\frac{\sqrt{n}}{2}, \pm\frac{\sqrt{n}}{2}, \pm\frac{\sqrt{n}}{2}\right)\]

The hypercube $S^2$ has 4 pairs of 3-D cubes, which form the boundary $\partial S^2$ of $S^2$.

Each 3-D cube has 6 faces. They form the boundary, $\partial S^2$, of $S^2$. There are $2\times 6$ such faces. They are depicted on the next page.

Each of them has exactly one point at its center. However, these points are not distinct. In fact, for the space-time coordinates $(t, x, y, z)$ of a center on one face, there exists the center of another face with the same coordinates.
For example, the point $P^t_+ \at t = \frac{\Delta t}{2}$ (first row, second column on page A.3) has the same coordinates, $(\frac{\Delta t}{2}, 0, \frac{\Delta x}{2}, 0)$, as the point $P^t_+ \at y = \frac{\Delta x}{2}$ (third row, second column). These points are therefore one and the same.

$\{P^t_+ \at t = \frac{\Delta t}{2}\} = \{P^t_+ \at y = \frac{\Delta x}{2}\} = (\frac{\Delta t}{2}, 0, \frac{\Delta x}{2}, 0)$

Consequently, the corresponding faces are (except for their opposite orientations) one and the same. For the 2-D domain where two different 3-D cubes meet in the 4-D spacetime.
It follows that the corresponding vector-valued moment of rotation
\[ \mathbf{\tau} (\mathbf{p}_+ - \mathbf{p}) \times \mathbf{e}_x \times \mathbf{R}^{\lambda \mu \lambda}_x \times \mathbf{d} \mathbf{a}_x \mathbf{d} \mathbf{a}_z \]
and
\[ \mathbf{\tau} (\mathbf{p}_- - \mathbf{p}) \times \mathbf{e}_x \times \mathbf{R}^{\lambda \mu \lambda}_x \times \mathbf{d} \mathbf{a}_x \mathbf{d} \mathbf{a}_z \]
also cancel.

Applying this reasoning to the remaining 23 pairs of faces one concludes that
\[ \mathbf{\tau} \times \mathbf{e}_x \times \mathbf{R}^{\lambda \mu \lambda}_x \times \mathbf{d} \mathbf{a}_x \mathbf{d} \mathbf{a}_z + \mathbf{\tau} (\mathbf{p}_- - \mathbf{p}) \times \mathbf{e}_x \times \mathbf{R}^{\lambda \mu \lambda}_x \times \mathbf{d} \mathbf{a}_x \mathbf{d} \mathbf{a}_z \]
+ 46 additional terms which cancel pairwise.\(^7\)

Note that each pair member does depend on \( \mathbf{p} \).

Upon rearranging these 48 surface integrals into 8 groups, with 6 terms for the 6 bounding faces of each of the
Eight cubes on page 13 are obtained.

cancelling terms?

\[ \frac{x^2}{5} \cdot \frac{(x-1)^2}{5} \cdot \frac{(x^2-1)}{5} \]

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