PAIR OF ACCELERATED FRAMES: A PERFECT INTERFEROMETER

Based on the second of two talks given June 23-24, 1997, at the 8th Marcel Grossmann Meeting, held in Jerusalem, Israel.

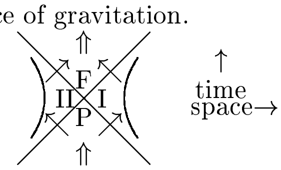
ULRICH H. GERLACH
Department of Mathematics, Ohio State University, Columbus, OH 43210, USA

The four Rindler quadrants of a pair of oppositely accelerated frames are identified as a (Lorentzian) Mach-Zehnder interferometer. The Rindler frequency dependence of the interference process is expressed by means of a (Lorentzian) differential cross section. The Rindler frequencies of the waves in the two acccelerated frames can be measured directly by means of a simple inertially moving detector.

Does there exist a purely quantum mechanical carrier of the imprints of gravitation? Mathematical aspects as well as the motivation for considering this question are discussed elsewhere in these proceedings. Here we summarizes some key physical aspects of this question.

Gravitation manifests itself by the imprints it leaves on the geodesic world lines of particles. It is a remarkable fact that even though particles may differ from one another in their mass and charge, the manner in which they reveal the presence of gravitation is totally independent of these intrinsic properties. It is stricly by means of their classical paths in spacetime that particles capture the imprints of gravitation. The mass and charge, i.e. internal composition, of these particles plays no role (uniqueness of free fall, "weak equivalence principle").

The last property is a key requirement dictated by the Dicke-Eotvos experiment. Adopting its extension to quantum mechanics ("imprints of gravitation are independent of a particle's Compton wave length and kinetic energy") has led us to pairs of oppositely accelerating ("Rindler") frames as the spacetime framework for measuring the presence of gravitation.



More precisely, the two accelerating frames serve as the two coherent legs of a Lorentzian version of the Mach-Zehnder interferometer. The two well-known pseudo-gravitational potentials in these frames serve as the two mirrors with 100% reflectivity. The two spacetime regions in the future F and the past P *near* the intersection ("bifurcation event") of the event horizons serve as "half-silvered" mirrors. A wave in P *far* from the bifurcation event enters the "interferometer" from P. One can show mathematically that near the bifurcation event the wave splits into two partial waves: one propagates across the past event horizon, enters Rindler Sector I, and gets reflected by its potential, the other does the analogous thing in Rindler Sector II. The two partial waves recombine *near* the bifurcation event in F and then leave the "interferometer" by proceeding towards a detector situated *far* from the bifurcation event. The detector (in F) is inertial, and it measures the Minkowski particle (or anti-particle) number.

The detector will record an interference pattern as follows: into accelerating

Rindler frame I place a dielectric slab so that it is static in (i.e. coaccelerating with) that frame. Because the refractive index differs from unity, the wave reflected in that frame will have suffered a phase shift. This phase shift depends on the thickness of the slab. Upon combining with the wave reflected in Rindler frame II, the phase shifted wave produces an alteration in the Minkowski particle count recorded by the detector in F. The interference pattern itself consists of the recorded particle count as a function of the thickness of the dielectric slab.

The determination of the refractive index with measurements based on an interference pattern illustrates the versatility of the "Lorentizian" Mach-Zehnder interferometer. Its two "legs" (Rindler Sectors I and II) not only (i) furnish a spacetime environment with a Cauchy hypersurface for the propagation of disturbaces governed by a wave equation, nor do they (ii) only serve as a nature-given arrangement for determining the effective parameters which characterize the propagation environment, but (iii) they also have a pair of effective ("pseudo-gravitational") potentials which provide a *diffractive aperture* through which waves diffract as they propagate from the past (P) to the future (F).

The interference pattern of a Lorentzian Mach-Zehnder interferometer is based on globally defined waves which are monochomatic, i.e. Lorentz boost invariant. One would have to consider such waves if the above "versatility" of the interferometer is to be realized. These wave modes have been exhibited explicitly [1], and they all satisfy the same wave equation, namely $\frac{\partial^2\psi}{\partial t^2} - \frac{\partial^2\psi}{\partial z^2} + k^2\psi = 0$, where $k^2 = k_x^2 + k_y^2 + m^2$, or

$$\left[\frac{1}{\xi^2}\frac{\partial^2}{\partial\tau^2} - \frac{1}{\xi}\frac{\partial}{\partial\xi}\xi\frac{\partial}{\partial\xi} + k^2\right]\psi = 0 \quad \text{in } I \text{ and } II \tag{1}$$

and

$$\left[-\frac{1}{\xi^2}\frac{\partial^2}{\partial\tau^2} + \frac{1}{\xi}\frac{\partial}{\partial\xi}\xi\frac{\partial}{\partial\xi} + k^2\right]\psi = 0 \quad \text{in } P \text{ and } F. \tag{2}$$

Even though the propagation of each wave mode from P to F constitutes a unique and separate Mach-Zehnder interference process, the nature-given interferometer for these processes consists of one and the same pair of Rindler frames, I and II. The only difference is the obvious one: the effective location (ξ) of the 100% reflective mirrors depends on the boost frequency of these modes. In fact, the effective proper separation between these two mirrors is $2\times(Rindler\ frequency)/k$ for a given mode. Beyond this distance the wave becomes evanescent.

Consider the propagation of a positive Minkowski frequency plane wave mode from P to F. It is a superposition of the globally defined monochromatic (i.e. pure Rindler frequency) wave modes, each one propagating from P to F and hence characterized by its own Mach-Zehnder interference process. This means that the propagation of a plane wave mode is equivalent to the simultaneous occurrence of all the corresponding Mach-Zehnder interference processes. There is one such process for each Rindler frequency. The interference is extremely delicate and of a very special kind. In F, each pair of monochromatic partial waves (having the same *spatial* Rindler frequency and coming from I and II respectively) has a phase relation such that an inertial detector, sensitive to the spatial frequency of these waves, will measure only particles, and no antiparticles.

Suppose there is a gravitational (or some other) disturbance in one or both of the two coherent legs of the interferometer. Such a disturbance will spoil the delicate interference. If the disturbance is Lorentz invariant, then the interference will be altered in a very simple and special way: only the phases in each of the pair of partial monochromatic wave modes from I and II will be altered. No mixing between waves of different Rindler frequency. Let $\delta_I(\omega)$ and $\delta_{II}(\omega)$ be the phase shift of the two wave amplitudes reflected from I and II. Then the *partial wave cross section* for the scattering of a plane wave by the Lorentz invariant disturbance is [2]

$$\frac{d\sigma}{d\omega} = \frac{1}{k \cosh\theta} \left\{1 + \left(\frac{\sin(\delta_I(\omega) - \delta_{II}(\omega))}{\sinh \pi\omega}\right)^2\right\} \tag{3}$$

Here $k \cosh\theta$ is the Minkowski frequency of the plane wave in P before it got scattered. This partial wave cross section expresses the interference pattern mentioned at the top of the previous page.

If the gravitational disturbance is *not* Lorentz invariant relative to the pair of accelerated frames, then the Mach-Zehnder interference processes will *not* be independent of one another, and there will be a mixing between the different Rindler frequencies.

Regardless of the detailed nature of the disturbance, the task of identifying it by means of its particle (or antiparticle) spectrum in F falls on the set of Rindler frequency selective particle detectors moving inertially in F. These detectors are discussed below. The information which they record is, roughly speaking, the intensity of the (Rindler frequency) fourier transform of the disturbance.

The physical justification for identifying the four Rindler sectors as a Lorentzian Mach-Zehnder interferometer depends on measuring the interfering Rindler frequency components from Sectors I and II. The waves having a specific Rindler frequency propagate across the future event horizon into Rindler sector F. There each of these waves is selectively measured by an *inertially moving* detector with a Fabry-Perot interference filter whose mirror separation increases with constant speed. Such a filter-detector combination in F responds only to *discrete Rindler frequencies*

$$\omega = \frac{\pi\ell}{\tanh^{-1}\beta} \quad , \quad \ell = 1, 2, \cdots \tag{4}$$

Here β is the relative speed of the two mirrors. These frequencies are the transmission resonances of the expanding Fabry-Perot cavity. The sharpness of these resonances, and hence the selectivity of the interference filter, depends on the reflectivity (< 1) of the Fabry-Perot mirrors. Waves with frequencies different from the discrete resonances get reflected, and hence do not interact with the detector.

Waves from Rindler I enter the cavity through one mirror, while those from Rindler II enter through the other. Coupled to the waves trapped between these two nearly, but not quite, completely silvered mirrors, the inertial detector lends itself to recording the Lorentzian Mach-Zehnder interference process.

References

[1] U.H. Gerlach, Phys. Rev. D 38, 514 (1988)

[2] U.H.Gerlach, "Scattering by a Pair of Oppositely Accelerated Dielectric Media", preprint (1997)