QUANTUM MECHANICAL CARRIER OF THE IMPRINTS OF GRAVITATION [a]

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We exhibit a purely quantum mechanical carrier of the imprints of gravitation by identifying for a relativistic system a property which (i) is independent of its mass and (ii) expresses the Poincare invariance of spacetime in the absence of gravitation. This carrier consists of the phase and amplitude correlations of waves in oppositely accelerating frames. These correlations are expressed as a Klein-Gordon-equation-determined vector field whose components are the "Planckian power" and the "r.m.s. thermal fluctuation" spectra. The imprints themselves are deviations away from this vector field.

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1 INTRODUCTION

Does there exist a purely quantum mechanical carrier of the imprints of gravitation? The motivation for considering this question arises from the following historical scenario: Suppose a time traveller from today visited Einstein in 1907, when he had the "happiest thought of his life", just in time before Einstein actually started on the path which led towards his formulation of gravitation (general relativity). Suppose that this time traveller told Einstein about relativistic quantum mechanics, about event horizons, and about Rindler's exact spacetime coordinatization of paired accelerated frames. Would that visitor from today have been able to influence the final form of the subsequent theory of gravitation? Put differently, how different would the course of history have been if Einstein had grafted relativistic quantum mechanics and Rindler spacetime onto the *roots* of gravitation instead of its trunk or branches?

2 THE EOTVOS PROPERTY

Relativistic quantum mechanics is a broad foundation of physics, but on the surface it seems to have no bearing on gravitation. Nevertheless, we shall choose this foundation as the vantage point for viewing gravitation. In order not to appear arbitrary, it is necessary that we remind ourselves of the key ideas that led Einstein to his formulation of gravitation and then assess how well they comply with relativistic quantum mechanics.

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2.1 Classical

Gravitation is characterized by the fact that every test particle traces out a world line which is independent of the parameters characterizing the particle, most notably its mass. Thus, cutting the mass of a planet into half has no effect on its motion in a gravitational field. No intrinsic mass parameter is needed to specify a particle's motion. The world line is determined entirely by the particle's local environment, not by the particle's intrinsic structure such as its mass. We shall refer to this structure independence as the "Eotvos property" [1].

This property is the key to making gravitation subject to our comprehension. This independence is far reaching, because, with it, the world lines have a remarkable property: They are probes that reveal the nature of gravitation without themselves getting affected by the idiosyncracies (e.g. mass) of the particles. The imprints of gravitation are acquired without the introduction of irrelevant features such as the mass of the system that carries these imprints. Put differently, the classical world lines of particles highlight a basic principle: Gravitation is to be identified by its essentials.

Einstein used this principle to acquire our understanding about gravitation within the framework of classical mechanics. His line of reasoning was as follows:

(1) He identified an "accelerated frame" as a one-parameter family of locally inertial (=free-float) frames [2]. The free-float nature of each of these frames he ascertained strictly within the purview of classical mechanics, namely by means of the straightness of all particle world lines (Newton's first law of motion) [3].

(2) Next he observed that the afore-mentioned Eotvos property of the particle world lines implies and is implied by the statement (equivalence principle) that the motion of particles, falling in what was thought to be a non-accelerated frame with gravity present, is physically equivalent to the motion of free particles viewed relative to an accelerated frame [4,5,6].

(3) With this observation as his staring point, he used Lagrangian mechanics to characterize gravitation in terms of the metric tensor [4,5,6], and then proceeded towards his theory of gravitation [7] along what in hindsight is a straight forward mathematical path [8]

However happy we must be about Einstein's gravitation theory, we must not forget that it rests on two approximations, which, although very fruitful, are approximations nevertheless. They are made in the very initial stages of Einstein's line of reasoning, namely in *what* he considered an "accelerated frame" and in *how* he used it.

First of all, in step (1) above, Einstein approximates an "accelerated frame" as one in which its future and past event (Cauchy) horizons are to be ignored. Such indifference is non-trivial. It results in neglecting the fact that (i) *a frame accelerating uniformly and linearly always has a* twin, *which moves into the opposite direction* and (ii) that it takes these *two* frames ("Rindler frames") to accomodate a Cauchy hypersurface.

Secondly, the geodesic world lines, the carriers of the imprints from which he constructs his theory of gravitation, are only classical approximations to the quantum mechanical Klein-Gordon wave functions. Their domain extends over all (the regions) of spacetime associated with the pair of accelerated frames. The world lines, by contrast, have domains which are strictly limited to the razor sharp classical particle histories.

It would have been difficult to argue with these approximations ninety years ago. However, the knowledge we have gained in the meantime would expose us to intellectual evasion if we were to insist on adhering to them in this day and age. Thus we shall not make them.

2.2 Quantum Mechanical

Even though we shall distance ourselves completely from these two approximations, we shall adhere to the above-mentioned principle in order to make gravitation comprehensible. This means that we shall bring the Eotvos property into the purview of quantum mechanics by insisting that we find a carrier of the imprints of gravitation which is *both* independent of the intrinsic mass of the carrier, *and* is purely quantum mechanical in nature.

This requirement turns out to be extremely restrictive, but not overly so.

In quantum mechanics, unlike in classical mechanics, the dynamics of a system is an issue separate from the measurement of its properties. Consequently, our formulation comes in two parts. There is the mathematical part which develops the dynamics of the carrier, and there is the physical part which describes how to measure the gravitational imprints with physically realizable apparatus. In this article we shall consider primarily the first part. A key aspect of the second is pointed out in the concluding section, while a more extensive sketch has been consigned to another article. [9].

3 ACCELERATION-INDUCED CARRIER OF THE IMPRINTS OF GRAVITATION

The most obvious candidate for such a carrier is the set of wave functions of a particle. They take over the role that the particle world lines occupy in classical mechanics. Indeed, in the classical limit, any world line can be recovered from the wave functions via the principle of constructive interference. We shall find that the imposition of the Eotvos property on the wave functions is not only extremely restrictive, but also forces us into viewing spacetime from a perspective which is very different from the customary one.

First recall the de Broglie relations. They relate the wavelength and the frequency of a particle's wave function to its momentum and energy, and to its mass. Next recall that, quite generally, the set of dynamical constants of motion of particles not only characterizes the particles, but also indicates the nature of the reference frame within which they move. In fact, there is a one-to-one correspondence between the constants of motion ("complete set of commuting observables") and the type of reference frame in which they are observed. Thus conservation of momentum and energy for all particles implies that they are observed in a free-float (inertial) reference frame. Consequently, if one chooses to describe the quantum dynamics of the particles in the momentum representation, then, by default, one has chosen to observe these particles relative to a free-float frame. This means that the choice of the representation characterized by a complete set of commuting observables (here the momentum and the energy) goes hand in hand with the customary view of observing spacetime from the perspective of free float frames.

In the momentum representation the wave function depends explicitly on the mass of the particle. Consequently, this wave function does not qualify as a carrier of the imprints of gravitation.

We shall now remedy this deficiency by exhibiting the quantum dynamics of the particle in a new representation relative to which the wave function does have the Eotvos property. The spacetime perspective of this representation consists of the familiar four Rindler sectors induced by two (noninertial) frames accelerating into opposite directions.

3.1 Relativistic Quantum Mechanics

Our starting point is non-trivial relativistic quantum mechanics as expressed by the Klein-Gordon equation

$$\frac{\partial^2\psi}{\partial t^2} - \frac{\partial^2\psi}{\partial z^2} + k^2\psi = 0, \tag{1}$$

where $k^2 = k_x^2 + k_y^2 + m^2$.

The objective is to deduce from this equation a carrier of the imprints of gravitation with the following three fundamental requirements:

1. The imprints must be carried by the evolving dynamics of a quantum mechanical wavefunction.

2. Even though the dynamical system is characterized by its mass m, the carrier and the imprints must *not* depend on this mass, i.e. the carrier must be *independent* of k^2. This requirement is analogous to the classical one in which the world line of a particle is independent of its mass.

3. In the absence of gravitation the carrier should yield measurable results (expectation values) which are invariant under Lorentz boosts and spacetime translations.

We shall now expand on these three requirements. In quantum mechanics the wave function plays the role which in Newtonian mechanics is played by a particle trajectory or in relativistic mechanics by a particle world line. That the wave function should also assume the task of carrying the imprints of gravitation is, therefore, a reasonable requirement.

Because of the Braginski-Dicke-Eotvos experiment, the motion of bodies in a gravitational field is independent of the composition of these bodies, in particular their mass. Consequently, the motion of free particles in spacetime traces out particle histories whose details depend only on the gravitational environment of these particles, not on their internal constitution (uniqueness of free fall, "weak equivalence principle"). Recall the superposition of different wave functions (states) of a relativistic particle yields interference fringes which do depend on the mass of a particle ("incompatibility between quantum and equivalence principle" [10]). If the task of these wave functions is to serve as carriers of the imprints of gravitation, then, unlike in classical mechanics, these interfering wave functions would do a poor job at their task: They would respond to the presence (or absence) of gravitation in a way which depends on the details of the internal composition (mass) of a particle. This would violate the simplicity implied by the Braginski-Dicke-Eotvos experiment. Thus we shall not consider such carriers. This eliminates any quantum mechanical framework based on energy and momentum eigenfunctions because the dispersion relation, $E^2 = m^2 + p_z^2 + p_y^2 + p_x^2$, of these waves depends on the internal mass m.

Recall that momentum and energy are constants of motion which imply the existence of a locally inertial reference frame. Consequently, requirement 2. rules out *inertial* frames as a viable spacetime framework to accomodate any quantum mechanical carrier of the imprints of gravitation. Requirement 2. also rules out a proposal to use the interference fringes of the gravitational Bohm-Aharanov effect to carry the imprints of gravitation [11]. This is because the fringe spacing depends on the rest mass of the quantum mechanical particle.

Requirement 3. expresses the fact that the quantum mechanical carrier must remain unchanged under the symmetry transformations which characterize a two-dimensinal spacetime. By overtly suppressing the remaining two spatial dimensions we are ignoring the requisite rotational symmetry. Steps towards remedying this neglect have been taken elsewhere [13].

We shall now exhibit a carrier which fulfills the three fundamental requirements. It resides in the space of Klein-Gordon solutions whose spacetime domain is that of a *pair* of frames accelerating into opposite directions ("Rindler frames"). These frames partition spacetime into a pair of isometric and achronally related Rindler Sectors I and II,

$$\left.\begin{array}{l} t - t_0 = \pm\xi \sinh\tau \\ z - z_0 = \pm\xi\cosh\tau \end{array}\right\} \quad \begin{array}{ll} + : & \text{"Rindler Sector I"} \\ - : & \text{"Rindler Sector II"} \quad . \end{array} \tag{2}$$

Suppose we represent an arbitrary solution to the K-G equation in the form of a complex two-component vector normal mode expansion

$$\left(\begin{array}{c} \psi_I(\tau,\xi) \\ \psi_{II}(\tau,\xi) \end{array}\right) = \int_{-\infty}^{\infty} \{a_\omega \left(\begin{array}{c} 1 \\ 0 \end{array}\right) + b_\omega^* \left(\begin{array}{c} 0 \\ 1 \end{array}\right)\} \sqrt{2|\sinh\pi\omega|}\frac{K_{i\omega}(k\xi)}{\pi} e^{-i\omega\tau} d\omega \equiv \int_{-\infty}^{\infty} \psi_\omega d\omega \quad . \tag{3}$$

This is a *correlated* ("entangled") state with two independent degrees of freedom. There is the *polarization* degree of freedom in addition to the spatial degree of freedom. The polarization degree of freedom has a two-dimensional space of states (C^2) spanned by two-spinors. The two components of a spinor refer to the wave amplitude at diametrically opposite events on a Cauchy hypersurface $\tau = constant$ in Rindler I and II respectively. The spatial ($0 < \xi < \infty$) degree of freedom has a state space ($\mathcal{H}$) which is ∞-dimensional and which is spanned by the scalar boost eigenfunctions

$$\phi_\omega = \sqrt{2|\sinh\pi\omega|}\;\frac{K_{i\omega}(k\xi)}{\pi}\;e^{-i\omega\tau} \quad , \tag{4}$$

solutions to the Rindler wave equation

$$\left[\frac{1}{\xi^2}\frac{\partial^2}{\partial\tau^2} - \frac{1}{\xi}\frac{\partial}{\partial\xi}\xi\frac{\partial}{\partial\xi} + k^2\right]\phi_\omega(\tau,\xi) = 0 \quad , \tag{5}$$

which is the equation obtained by applying the coordinate transformation Eq.(2) to the Klein-Gordon Eq.(1).

3.2 Geometry of the Space of Solutions

The representation (3) puts us at an important mathematical juncture: We shall forego the usual picture of viewing this solution as an element of Hilbert space ($C^2 \otimes \mathcal{H}$) with the Klein-Gordon inner product,

$$
\begin{aligned}
&\frac{i}{2}\int_{\infty}^{0} \psi_{II}^{*} \overset{\leftrightarrow}{\frac{\partial}{\partial\tau}} \psi_{II} \frac{d\xi}{\xi} + \frac{i}{2}\int_{0}^{\infty} \psi_{I}^{*} \overset{\leftrightarrow}{\frac{\partial}{\partial\tau}} \psi_{I} \frac{d\xi}{\xi} \\
&\quad = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (a_\omega^* b_\omega) \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{pmatrix} a_{\omega'} \\ b_{\omega'}^* \end{pmatrix} \frac{i}{2}\int_{0}^{\infty} \phi_\omega^* \overset{\leftrightarrow}{\frac{\partial}{\partial\tau}} \phi_{\omega'} \frac{d\xi}{\xi} \\
&\quad = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (a_\omega^* b_\omega) \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{pmatrix} a_{\omega'} \\ b_{\omega'}^* \end{pmatrix} \frac{\omega}{|\omega|}\delta(\omega-\omega')\, d\omega d\omega' \quad \begin{matrix}\text{(inner product} \\ \text{for } C^2 \otimes \mathcal{H}\ )\end{matrix} \\
&\quad \equiv \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \langle \psi_\omega, \psi_{\omega'} \rangle \, d\omega d\omega' \quad . \qquad (6)
\end{aligned}
$$

Instead, we shall adopt a qualitatively new and superior viewpoint. *Each Klein-Gordon solution is a spinor field over the Rindler frequency domain.* This is based on the vector bundle $C^2 \times R$. Here C^2 is the complex vector space of two-spinors, which is the fiber over the one-dimensional base manifold $R = \{\omega : -\infty < \omega < \infty\}$, the real line of Rindler frequencies in the mode integral, Eq.(3).

We know that one can add vectors in the *same* vector (fiber) space. However, one may not, in general, add vectors belonging to different vector spaces at different ω's. The exception is when vectors in different vector spaces are *parallel*. In that case one may add these vectors. The superposition of modes, Eq.(3), demands that one do precisely that in order to obtain the two respective total amplitudes of Eq.(3). In brief, we are about to show that that *the linear superposition principle determines a unique law of parallel transport*.

The mode representation of Eq.(3) determines two parallel basis spinor fields over R,

$$\left\{\begin{pmatrix} 1 \\ 0 \end{pmatrix} \phi_\omega : \; -\infty < \omega < \infty\right\} \text{ and } \left\{\begin{pmatrix} 0 \\ 1 \end{pmatrix} \phi_\omega : \; -\infty < \omega < \infty\right\} \quad , \qquad (7)$$

one corresponding to "spin up" (ψ has zero support in Rindler II), the other to "spin down" (ψ has zero support in Rindler I). This parallelism is dictated by the superposition principle, Eq.(3): The total amplitude at a point (τ, ξ) in Rindler I (resp. II) is obtained by adding all the contributions from Rindler I (resp. II) only.

One can see from Eq.(6) that, relative to this basis, the fiber metric over each ω is given by

$$(a_\omega^* b_\omega) \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{pmatrix} a'_\omega \\ b_{\omega'}^* \end{pmatrix} = a_\omega^* a_\omega - b_\omega b_\omega^* ; \quad -\infty < \omega < \infty \quad . \qquad (8)$$

It is evident that the law of parallel transport defined by Eq.(7) is *compatible* with this (Klein-Gordon induced) fiber metric. This is because the two parallel vector fields, which are represented by

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 0 \\ 1 \end{pmatrix}; \quad -\infty < \omega < \infty \tag{9}$$

relative to the basis (7), are orthonormal with respect to the the fiber metric Eq.(8) at each point ω of the base space R. It is not difficult to verify that these two spinor fields are (Klein-Gordon) orthonormal in each fiber over R. The spinor field

$$\{\begin{pmatrix} a_\omega \\ b^*_\omega \end{pmatrix} : \; -\infty < \omega < \infty\} \tag{10}$$

is a section of the fiber bundle $C^2 \times R$ and it represents a linear combination of the two parallel vector fields. It is clear that there is a *one-to-one correspondence between* $\Gamma(C^2 \times R)$*, the* ∞*-dimensional space of sections of this spinor bundle, and the space of solutions to the Klein-Gordon equation.*

3.3 Quantum Mechanical Carrier of Gravitational Imprints

Our proposal is to have each spinor field serve as a carrier of the imprints of gravitation: A gravitational disturbance confined to, say, Rindler I or II would leave its imprint on a spinor field at $\tau = -\infty$ by changing it into another spinor field at $\tau = +\infty$.

We know that in the absence of gravitation each of the positive and negative Minkowski plane wave solutions evolves independently of all the others. This scenario does not change under Lorentz boosts and spacetime translations. This is another way of saying that the system described by these solutions is Poincare invariant. Will the proposed carriers comply with this invariance, which is stipulated by fundamental requirement 3.? To find out, consider a typical plane wave. Its spinor representation (3) is

$$\begin{bmatrix} e^{-i(t-t_0)k\cosh\theta + i(z-z_0)k\sinh\theta}\big|_I \\ e^{-i(t-t_0)k\cosh\theta + i(z-z_0)k\sinh\theta}\big|_{II} \end{bmatrix} = \int_{-\infty}^{\infty} e^{i\omega\theta} \begin{bmatrix} e^{\pi\omega/2} \\ e^{-\pi\omega/2} \end{bmatrix} \frac{K_{i\omega}(k\xi)}{\pi} e^{-i\omega\tau} d\omega \quad . \tag{11}$$

This is a state with 100% correlation between the boost energy and the polarization ("spin") degrees of freedom. In recent years such states have been called "entangled" states[12]. Suppose that for each boost energy we determine the normalized Stokes parameters of this polarization, i.e. the expectation values of the three modified Pauli "spin" matrices

$$\vec{\sigma} : \{\sigma_1, \sigma_2, \sigma_3\} = \left\{ \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -i \\ -i & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \right\} \quad . \tag{12}$$

This is a three-dimensional vector field over the base manifold R, and is given by [13]

$$\frac{\langle \psi_\omega, \vec{\sigma}\psi_{\omega'} \rangle}{\langle \psi_\omega, \psi_{\omega'} \rangle} = \pm \left(\sqrt{N(N+1)}, 0, \frac{1}{2} + N \right); \quad N = (e^{2\pi\omega} - 1)^{-1}; \quad -\infty < \omega < \infty \tag{13}$$

In compliance with requirements 2. and 3., this vector field is (a) *independent of the particle mass* and (b) the same for *all* positive (negative) Minkowski plane wave modes, a fact which expresses its Poincare invariance. The presence of *gravitation would leave its imprints by producing characteristic alterations in this vector field.*

4 SUMMARY

4.1 This Article

Equation (13) is the center of a constellation consisting of the following three results:

The main result is the recognition of the fact that a Klein-Gordon charge has a property which displays the Poincare invariance of Minkowski spacetime *without involving its specific mass.* This property is the set of expectation values given by Eq.(13). The presence of gravitation is expressed by distortions of this mass-independent vector field. This property is a quantum mechanical extension of the principle familiar from classical mechanics, and it is dictated by the Braginski-Dicke-Eotvos experiment, that the set of particle trajectories serve as the carrier of the imprints of gravitation.

The second result is the fact that Eq.(13) are (twice) the expectation values of the "spin" component operators

$$\mathbf{L}: \ L_1 = \frac{\sigma_1}{2},\ L_2 = \frac{\sigma_2}{2},\ L_3 = \frac{\sigma_3}{2} \quad . \tag{14}$$

Their commutation relations

$$\begin{aligned} [L_1, L_2] &= -iL_3 \\ [L_2, L_3] &= iL_1 \\ [L_3, L_1] &= iL_2 \quad , \end{aligned} \tag{15}$$

are those of the symmetry group $SU(1,1)$, which is precisely the invariance group of the fiber metric, Eq.(8). The fact that $SU(1,1)$ is the invariance group of the single charge system extends into the classical regime: If one interprets the Klein-Gordon wave function as a classical field, then this $SU(1,1)$ symmetry gives rise [13] to a *conserved vectorial "spin"*, whose density is given by one half the expectation value of σ, as in the numerator of Eq.(13).

The third result is as obvious as it is noteworthy: The components of the vector field coincide with the "Planckian power" and the "r.m.s. thermal fluctuation" spectra, in spite of the fact that we have not made any thermodynamic assumptions. In fact, we are considering only the quantum mechanics of a single charge, *or* the Klein-Gordon dynamics of a classical wave field. That these two spectra also arise within the framework of a strictly classical field theory, has already been observed in [14]. We would like to extend this observation by pointing out that in the absence of gravitation these two spectra express the invariance of spacetime under translations and Lorentz trasformations (requirement 3., Poincare invariance)

4.2 The Wider Perspective

We comprehend gravitation in two stages. First we identify the agent which carries the imprints of gravitation. In Newton's formulation this agent is the set of particle trajectories, in Einstein's formulation the set of particle world lines, and in the quantum formulation the set of correlations between the wave amplitudes in a pair of oppositely accelerating Rindler frames.

In Newton's formulation the imprints consist of the bending of the particle trajectories, in Einstein's formulation they consist of the deviations of the geodesic world lines, and in the quantum formulation they consist of the deviations of the correlations away from the "Planckian" and the "fluctuation" spectral values given by Eq.(13).

The second stage of our comprehension consists of relating these imprints to the source of gravitation. In Newton's formulation this relation is the Poisson equation, in Einstein's formulation the field equations of general relativity, and in the quantum formulation we do not know the answer as yet.

The equations for gravitation are an expression of the relation between the properties inside a box and the resulting geometrical imprints of gravitation on the surface of the box which surrounds the matter source of gravitation. In the Newtonian theory the mass inside the box is proportional to the total amount of gravitational force flux through the surface of the box. In Einstein's theory the amount of energy and momentum in the box is proportional to the total amount of moment of rotation on the surface of the box[16,17].

Alternatively, if the box is swept out by a coplanar collimated set of moving null particles (e.g. neutrino test particles), then, upon moving from one face of the box to the other, the neutrino pulse gets focussed by an amount which is proportional to the amount of matter inside the swept-out box. This proportionality, when combined with energy-momentum conservation, is expressed by the Einstein field equations[18]. In this formulation the directed neutrino pulse is a classical (i.e. non-quantum mechanical) carrier of the imprints of gravitation, and the amount by which the pulse area gets focussed is the gravitational imprint which is also classical. By letting the neutrino pulse go into various directions, one obtains the various components of the Einstein field equations.

It is interesting that the logical path from the classical imprints to these field equations consists of a temporary excursion into the quantum physics relative to an accelerated frame. This excursion starts with the demand that one describe the state of the matter inside the box relative to the frame of a uniformly accelerated observer. The acceleration of this frame is to be collinear with the motion of the neutrino pulse. If one complies with this demand, then the matter passing through the neutrino pulse (event horizon) area is a flow of heat energy relative to the accelerated frame. The temperature is the acceleration temperature given by the Davies-Unruh formula. This permits one to assign an entropy to the matter inside the box swept out by the neutrino pulse. Consequently, the Rindler (boost) heat energy in the box is the product of the matter entropy times the acceleration temperature.

Finally, the Einstein field equations follow from the Bekenstein hypothesis[19] that the entropy be proportinal to the change of the area of the neutrino pulse area as it passes through the box.

It is obvious that this deduction of the field equations from the Bekenstein hypothesis is only a temporary excursion into the quantum physics relative to an accelerated frame. Indeed, even though the proportionality between entropy and area consists of the squared Planck-Wheeler length, reference to Planck's constant $\hbar$ gets cancelled out by the Davies-Unruh temperature in the product which makes up the Rindler heat flux, the source for the Einstein field equations. Consequently, the disappearance $\hbar$ from the heat flux guarantees that $\hbar$ will not appear in the Einstein field equations.

It seems evident that a quantum mechanical comprehension of gravitation must start with a purely quantum mechanical carrier of its imprints.

5 CONCLUDING REMARK

Recall that in quantum mechanics, unlike in classical mechanics, the problem of the dynamics of a system and the measurement of its properties are two different issues. The dynamics is governed by a differential equation, in our case the system's wave equation, Eq.(1). The measured properties are expressed by the expectation values of the appropriately chosen operators.

It is obvious that our present treatment of these two issues has been rather lopsided and we must address how to measure the imprints of gravitation with physically realizable apparatus. At first this seems like an impossibple task.

Consider the fact that that the quantum dynamics is governed by the evolution of the Klein-Gordon wave function in the two Rindler frames, Eq.(2), which (i) are accelerating *eternally* and (ii) are *causally disjoint*. How can one possibly find a physically realistic observer who can access such frames? It seems one is asking for the metaphysically impossible: an observer which can accelerate eternally and can be in causally disjoint regions of spacetime. There simply is no such observer!

However, we are asking the wrong questions because we have been ignoring the two remaining Rindler sectors P and F.

$$\left.\begin{aligned} t - t_0 &= \pm\xi\cosh\tau \\ z - z_0 &= \pm\xi\sinh\tau \end{aligned}\right\} \qquad \begin{aligned} +&: \text{ “Rindler Sector F”} \\ -&: \text{ “Rindler Sector P”} \end{aligned} \tag{16}$$

If one includes them into the identification of the carrier of gravitational imprints, then our formulation leads to an astonishing conclusion: the four Rindler sectors I, II, P and F together form a *nature-given interferometer*, to be precise, the *Lorentzian* version of a Mach-Zehnder interferometer [15]. The pair of oppositely accelerating frames I and II accomodate the pair of widely separated coherent beams, and the two respective pseudo-gravitational potentials serve as the two perfectly reflecting mirrors. The two Rindler sectors P and F serve as the respective half-silvered mirrors: P acts as a beam splitter of radiation coming in from the past, while F is where the two reflected beams interfere to produce a wave propagating into the future.

Space limitations demand that we consign the task of elaborating on this terse description to another article [9]. The point is that the quantum mechanical carrier, Eq.(13), has a firm physical foundation while at the same time it exhibits the Eotvos property of being independent of the intrinsic mass of the quantum system.

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11. J.S.Anandan, Nuov. Cim. **53 A**, p.221(1979); also J.S.Anandan in B.L.Hu, M.P.Ryan, and C.V.Vishveshwara (eds.), *Directions in General Relativity, Volume 1*, (Cambridge University Press, 1993) p.10
12. A. Peres, *Quantum theory: Concepts and Methods*, (Kluwer Academic, Dordrecht, 1993)ch. 5
13. U.H.Gerlach in R.T.Jantzen and G.M.Keiser (eds.), *The Seventh Marcel Grossmann Meeting, Part B*, World Scientific Publishing Co. (1996), *ibid.* International Jour. of Mod. Phys. **11**, p.3667 (1996). This last reference contains three errata: (i) On page 3668, line 6, the word "existent" should be substituted for the word "existence". Without this substitution the parenthet-

ical definition becomes meaningless. (ii) The whole paragraph in the middle of page 3670 is a misplaced version of the Caption to Figure 1. Consequently, this paragraph should be deleted. (iii) The corrected sentence at the very end of Section 7 (on page 3686) should read "... the spinor transformation which relates Eq.(5.15) to Eq.(5.18)."

14. K. Srinivasan, L. Sriramkumar and T. Padmanabhan, "Plane waves viewed from an accelerated frame", Phys. Rev. **D** Nov.15 (1997); http://xxx.lanl.gov preprint gr-qc/9606017
15. See, e.g., M. Born and E. Wolf, *Principles of Optics*, sixth edition, (Pergamon Press, Elmsford, N.Y., 1980), p.312
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