RADIATION FROM VIOLENTLY ACCELERATED BODIES*

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A determination is made of the radiation emitted by a linearly uniformly accelerated uncharged dipole transmitter. It is found that, first of all, the radiation rate is given by the familiar Larmor formula, but it is augmented by an amount which becomes dominant for sufficiently high acceleration. For an accelerated dipole oscillator, the criterion is that the center of mass motion become relativistic within one oscillation period. The augmented formula and the measurements which it summarizes presuppose an *expanding* inertial observation frame. A *static* inertial reference frame will not do. Secondly, it is found that the radiation measured in the expanding inertial frame is received with 100 percent fidelity. There is no blueshift or redshift due to the accelerative motion of the transmitter. Finally, it is found that a pair of coherently radiating oscillators accelerating (into opposite directions) in their respective causally disjoint Rindler-coordinatized sectors produces an interference pattern in the expanding inertial frame. Like the pattern of a Young double slit interferometer, this Rindler interferometer pattern has a fringe spacing which is inversely proportional to the proper separation and the proper frequency of the accelerated sources. The interferometer, as well as the augmented Larmor formula, provide a unifying perspective. It joins adjacent Rindler-coordinatized neighborhoods into a single spacetime arena for scattering and radiation from accelerated bodies.

I. INTRODUCTION

The emission or the scattering of light from localized sources is the most effective way for information to be transferred to the human eye, the window to our mind. We can increase the size of this window, with specialized detectors, or systems of detectors. They intercept the information, record and re-encode it, before passing it on to be assimilated and digested by our consciousness.

The spacetime framework for most physical measurements, in particular those involving radiation and scattering processes, consists of inertial frames, or frames which become nearly inertial by virtue of the limited magnitude of their spatial and temporal extent. Indeed, the asymptotic "in" and "out" regions of the scattering matrix as well as the asymptotic "far-field" regions of a radiator reflect the inertial nature of the spacetime framework for these processes.

Should one extend these processes to accelerated frames? If so, how? Let us delay answering the first question and note that Einstein, in his path breaking 1907 paper [1], gave us the answer to the second: View an accelerated frame as a sequence of instantaneous locally inertial frames. Thus a scattering (or any other physical) process observed relative to a lattice of accelerated clocks and equally spaced detectors can be understood in terms of the lattice of inertial clocks and equally spaced inertial detectors [2] of one or several of these instantaneous locally inertial frames. Accelerated frames seem to be conceptually superfluous! Acceleration can always be transformed away by replacing it with an appropriate set of inertial frames. To make observations relative to an accelerated frame comprehensible, formulating them in terms of a sequence of instantaneous inertial frames seems (at first sight) to be sufficient.

The introduction of these inertial frames into physics was one of the two historical breakthroughs [3] for Einstein, because mathematically they are the tangent spaces, the building blocks from which he built general relativity.

However, characterizing an accelerated frame as a one-parameter family of instantaneous Lorentz frames was only an approximation, as Einstein himself points out explicitly [1] in his 1907 article. The approximation consists of the fact that the Lorentz frames *never have relativistic velocities* with respect to one another. Thus Einstein approximated a hyperbolic world line in I of Figure 1 by replacing it with a finite segment having the approximate shape of a parabola. If Einstein had not made this assumption, then he would have found immediately that associated with every uniformly linearly accelerated frame there is a *twin* moving into the opposite direction, and causally disjoint from the first. Nowadays these twins are called Rindler sectors I and II as in Figure 1. Thus, *linearly uniformly accelerated frames always come in pairs*, which (a) are causally disjoint and (b) have lightlike boundaries, their past and future horizons.

*Published in Phys. Rev. D **64**, 105004 (2001)

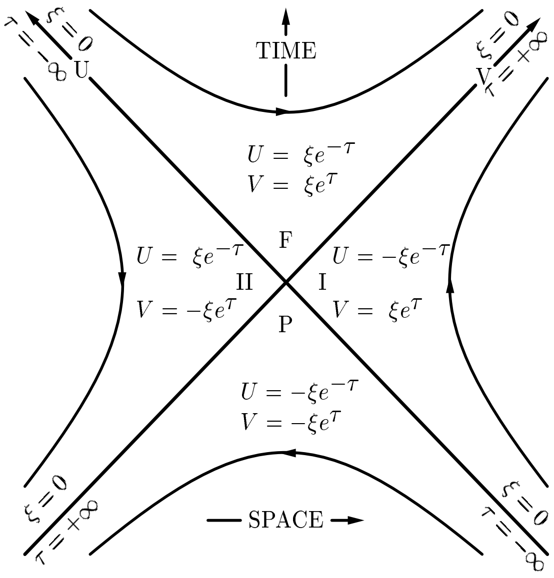

FIG. 1. Acceleration-induced partitioning of spacetime into the four Rindler coordinatized sectors. They are centered around the reference event (t_0, z_0) so that $U = (t - t_0) - (z - z_0)$ and $V = (t - t_0) + (z - z_0)$ are the retarded and advanced time coordinates for this particular quartet of Rindler sectors.

These two Rindler coordinatized sectors together with their past P and future F form a *double-slit interferometer* [4] relative to a spatially homogeneous but expanding coordinate frame. The two Rindler sectors I and II comprise the double slit portal through which wave fields propagate from P to F. During this process the wave field interacts with sources, which due to their acceleration, are confined to, say, Rindler sectors I and/or II. The interference between the waves coming from these two sectors is observed in F. There the field amplitude is sampled in space and in time.

Consider the field which is due to accelerated sources in I or in II. A single inertial radio receiver which samples the field temporally is confronted with a metaphysically impossible task: Track and decode a signal with a Doppler chirp (time dependent Doppler shift) whose phase is logarithmic in time. The longer and more violent the acceleration of the source, the more pronounced the initial blueshift and/or the final redshift at the receiver end. Tracking the amplitude and the phase of such a chirped signal becomes a debilitating task for any receiver.

Suppose, however, the field gets intercepted by a set of mutually receding radio receivers. If they, in concert, sample the field spatially at a single instant of "synchronous" time, then there is no Doppler chirp whatsoever. An accelerated source which emits a sharp spectral line will produce an equally sharp spectral line in the spatial Fourier domain of the sampled space domain (in Figure 1: $UV = \xi^2 = const.$) of the expanding set of radio receivers. In brief, *a signal emitted by an accelerated point source is intercepted by a set of mutually receding phased radio receivers with 100 % fidelity.* We shall refer to this result as the *fidelity property* of Rindler's spacetime geometry.

The physical reason for this result is given in Section III, the mathematical formulation in Section VI A.

The application of the fidelity property to the power emitted from an accelerated dipole oscillator is given in Section VII. This application consists of Larmor's formula [5] augmented due to the fact that the oscillator is in a state of uniform acceleration.

The fidelity property applies to the radiation from a source accelerated in Rindler I as well as to a source accelerated in Rindler II. If the two sources have the same frequency and are coherent, then the phased array of radio receivers measures an interference pattern which is mathematically indistinguishable from that due to a standard double slit. This result is spelled out in Section VI C.

It is worth while to reiterate that the fidelity property and its two applications are statements about the Rindler coordinate neighborhoods considered jointly, with the event horizons, $\xi = 0$, integral building blocks of these concepts. Some workers in the field [6], who view spacetime only in terms of "coordinate patches" or "coordinate charts" (i.e. comply with Einstein's approximation mentioned above), tend to compare the locus of events $\xi = 0$ in Fig. 1 to the coordinate singularity at the North Pole of a sphere or the origin of the Euclidean plane. Such a comparison leads to a pejorative assessment of Rindler's coordinatization as "imperfect","singular", or "poor" at $\xi = 0$ [6]. This is unfortunate. As a result, this comparison diverts attention from the fact that (1) waves from I and II interfere in F and that (2) as a consequence, the resulting interference patterns serve as a natural way of probing and measuring scattering and/or radiative sources as well as gravitational disturbances in regions I and II.

The Rindler double-slit opens additional vistas into the role of accelerated frames I and II. They accommodate causally disjoint but correlated radiation and scattering centers whose mutually interfering radiation is observed and measured in F. These measurements are mathematically equivalent to having two accelerated observers in Rindler I and II respectively. From these measurements one can reconstruct in all detail the location and temporal evolution of all accelerated radiation sources. The aggregate of these sources comprises what in Euclidean optics is called an object, one in Rindler I the other in Rindler II. What is observed in F is the interference of two coherent diffraction patterns of these two objects. These measurements are qualitatively different from those that can be performed in any *static* inertial frame. They yield the kind of information which can be gathered only in accelerated frames with event horizons. One of the virtues of the Rindler double-slit interferometer is that it quite naturally avoids an obvious metaphysical impossibility [7], namely, have accelerating observers in Rindler sectors I and II which (a) have the physical robustness to withstand the high (by biological-technological standards) acceleration and/or (b) the longevity and the propulsion resources to co-accelerate for ever and never cross the future event horizon.

From the perspective of implementing measurements, the Rindler double-slit has advantages akin to those of a Mach-Zehnder interferometer [9]: it permits an interferometric examination of regions of spacetime whose expanse is spacious enough to accommodate disturbances macroscopic in extent, and it permits one to achieve this feat without putting the measuring apparatus into harm's way. However, in order to use the Rindler interferometer as a diagnostic tool one must first have the necessary conceptual infrastructure. This article provides four of its ingredients:

- Expanding free float frame
- Fidelity property of the Rindler spacetime geometry
- Augmented Larmor formula
- Double slit interference due to a pair of accelerated sources

Nomenclature: This articlae uses repeatedly the words "Rindler sector", "Rindler spacetime", etc. This is verbal shorthand for "Rindler coordinatized sector", "Rindler coordinatized spacetime" etc. The implicit qualifier "coordinatized" is essential because, without it, "Rindler sector/spacetime" would become a mere floating abstraction, i.e. an idea severed from its observational and/or physical basis.

II. EXPANDING INERTIAL OBSERVATION FRAME

Fundamentally all of physics, including the physics of spacetime, is based on measurements. The class of measurements we shall focus on are those made by "recording clocks" [8] in a state of "free float". The meaning of a "recording clock" is that each one of them consists of

(i) a clock oscillator which controls the clock and has a standard frequency,

(ii) a transmitter whose emission frequency is controlled by this oscillator,

(iii) a receiver capable of measuring the emitted radiation from the other recording clocks, even if there is a Doppler shift, and

(iv) memory chips which can hold data acquired by the receiver.

Thus each clock is constructed like one of the GPS (Global Positioning System) units orbiting the earth. Assuming no gravitation, one says that the aggregate of recording clocks is in a state of "free float" (inertial motion) if the relative Doppler shift between each pair of such clocks is fixed and constant in time. There is *no Doppler chirp.* In other words, each recording clock measures (and stores in its memory) spectral lines which are sharp. The sharper the measured spectral lines the more closely the recording clocks are in a state of free float.

From this swarm of freely floating recording clocks one now forms, by a process of measurement omission, an equivalence class called an "expanding free float frame".

A. Construction

The formation of this concept is achieved by using one of the clocks, say R, as a reference clock which measures and collects two kinds of data about all the other clocks, say $A, B, C, \cdots$: Doppler shifts and instantaneous distance of $A, B, C, \cdots$ from R.

Doppler shift measurements are frequency measurements. The emission frequencies of all clocks $A, B, C, \cdots$ are the same, say ω_0. Consequently, the frequencies $\omega_A, \omega_B, \omega_C, \cdots$ received and measured by R yield the corresponding Doppler shift factors

$$k_A = \frac{\omega_A}{\omega_0} \equiv \exp(-\tau_A), \;\; k_B = \frac{\omega_B}{\omega_0} \equiv \exp(-\tau_B), \;\; k_C = \frac{\omega_C}{\omega_0} \equiv \exp(-\tau_C), \;\; \cdots$$

for the respective clocks. From these Doppler shift factors one obtains the relative velocities

$$v_A = \frac{\frac{1}{k_A} - k_A}{\frac{1}{k_A} + k_A} \equiv \tanh \tau_A, \;\; v_B = \frac{\frac{1}{k_B} - k_B}{\frac{1}{k_B} + k_B} \equiv \tanh \tau_B, \;\; etc.$$

between R and the respective clocks.

The second kind of measurement is the instantaneous separation. Suppose at some instant of time, say t_R, R measures the distances $d_A, d_B, d_C, \cdots$ between R and the clocks $A, B, C, \cdots$. One way of doing this is to have R operate his transmitter and receiver as a radar device.

We now say that A is equivalent to B, or more briefly $A \sim B$, if

$$\frac{d_A}{v_A} = \frac{d_B}{v_B} \equiv \xi(t_R) \; . \tag{1}$$

It follows that

1. $A \sim B$ implies $B \sim A$ and
2. $A \sim B$ and $B \sim C$ imply $A \sim C$.

By retaining the distinguishing property, the equality of the ratios, Eq.(1), while omitting reference to the particular measurements $d_A, d_B, d_C, \cdots$ and $k_A, k_B, k_C, \cdots$ one forms an equivalence class, the concept "expanding inertial frame". It should be noted that the reference clock R is always a member of this equivalence class. This is because, with $d_R = 0$ and $v_R = 0$, the equivalence condition

$$\frac{d_R}{v_R} = \frac{d_B}{v_B}$$

is satisfied trivially.

The properties of this equivalence class do not depend on the time at which R makes the distance measurements and hence not on the value of the ratio $\xi(t_R)$. Indeed, if instead of t_R that time had been , say t'_R, then the corresponding distance measurements would be $d'_A, d'_B, d'_C, \cdots$, then one would still have a set of equal ratios

$$\frac{d'_A}{v_A} = \frac{d'_B}{v_B} \equiv \xi(t'_R) \; ,$$

which would yield the same equivalence class of free float clocks.

The purpose of an inertial reference frame is for a physicist/observer to use its recording clocks to measure time and space displacements. These measurements consist of establishing quantitative relationships (typically via counting) to a standard which serves as a unit. For a time measurement the unit is the standard interval between any two successive ticks of a clock. For a space measurement the unit is the (logarithm of the) standard Doppler shift factor between any pair of nearest neighbor clocks. Thus the array of clocks forms a lattice which is periodic but is expanding uniformly: the recession velocity between any neighboring pair of clocks is one and the same. This periodicity is an obvious but tacit stipulation in what is meant by "expanding inertial frame". Because of this property any one of the recording clocks $A, B, C, \cdots$ can play the role of the reference clock R, which is to say that the equivalence relation, Eq.(1), is independent of the choice of R.

The two kinds of measurements which gave rise to the equivalence relation between recording clocks also serve to synchronize their operations. Every clock synchronizes itself to its nearest neighbor by setting its own clock reading to the ratio of (i) the nearest neighbor distance and (ii) the Doppler shift determined velocity. Thus the common ratio, Eq.(1), is the synchronous time common to all recording clocks. This common time has an obvious interpretation: The straight-line extensions into the past of all clock histories intersect simultaneously in a common point. This is a singular event, which corresponds to $\xi = 0$. The common synchronous time of these clocks is the elapsed proper time since then. However, it is obvious that this singular event is irrelevant for the definition of the expanding inertial frame. What is relevant instead is the ability of the recording clocks to measure Doppler shifts and distances, which presupposes that $\xi \neq 0$. In fact, these clocks might not even have existed until they performed their measurements.

Having constructed the spacetime measuring apparatus, we indicate in general terms how to make spacetime measurements of particles and fields.

B. Measurements of Particles and Fields

The mechanical measurements by a physicist/observer of the spacetime properties of a classical particle consists of (a) identifying which clock detects the existence of the particle in what interval of synchronous time and (b) determining the particle's velocity by measuring its Doppler shift. The first is a counting process in space and time, the second is a counting process in temporal frequency space.

The wave mechanical measurements of the spacetime properties of a classical electromagnetic (e.m.) field consist of using the expanding set of recording clocks to form a *phased array* of mutually receding receivers. This array samples the e.m. field at the locations of the recording clocks at regular intervals of synchronous time. The phased array mode involves all clocks at once and thus provides a record of the magnitude and spatial phase of the e.m. field. By repeating this procedure at temporal intervals controlled by the synchronized ticking of the clocks, one obtains a sampled historical record of the magnitude of the field and its temporal phase.

III. TRANSMISSION FIDELITY

The transfer of information from a transmitter to a receiver, or a system of receivers, depends on being able to establish a one to one correspondence between (i) the phase and amplitude of the e.m. source and (ii) the e.m. signals detected by the observer who mans the receiver(s) in his frame of reference. For a localized source with a straight worldline that frame is *static* and inertial. For a source with a hyperbolic worldline as in Figure 1, it is *expanding* and inertial. The e.m. signals are detected by having the recording clocks sample and measure the e.m. field at any fixed synchronous time $\xi > 0$. Except for a ξ-dependent amplitude and domain shift, these measured field values (along the spatial domain $-\infty < \tau < \infty$) are precisely the values of the current source (along the temporal domain $-\infty < \tau' < \infty$) of the accelerated transmitter [10]. Geometrically one says that the transmitter signal-function, whose domain is a timelike hyperbola in Rindler sector I, coincides in essence with the receiver signal function whose domain is a spacelike hyperbola in Rindler sector F. Thus, if the signal-function is monochromatic at the transmitter end, then so is the signal function on the spatial domain at the receiver end. *There is no chirp* (changing wave length) *in the spatial wave pattern in the expanding inertial frame.*

One arrives at that conclusion by verifying it for wave packets, i.e. for narrow but finite pulses of nearly monochromatic radiation, which make up the e.m. signal. Thus consider a uniformly and linearly accelerated transmitter. The history of its center of mass is represented by a timelike hyperbola in, say, Rindler sector I in Figure 1.

Let us have this single transmitter emit *two successive* pulses which have the same mean frequency and require that they be received at the same synchronous time ξ by *two adjacent* recording clocks in F. One has therefore two well-defined emission-reception processes,

$$(\tau'_A, \xi') : A(\tau'_A, \xi') \xrightarrow{\text{world history of pulse } A} (\xi, \tau_A) : A(\xi, \tau_A) \tag{2}$$

and

$$(\tau'_B, \xi') : B(\tau'_B, \xi') \xrightarrow{\text{world history of pulse } B} (\xi, \tau_B) : B(\xi, \tau_B) \; . \tag{3}$$

Each starts with a pulse emitted by the accelerated transmitter in Rindler sector I and ends with the pulse's reception by an inertial recording clock in Rindler sector F. Both processes end with the simultaneous reception of these pulses in the expanding inertial reference frame. Among its recording clocks there are precisely two, labelled by τ_A and τ_B, which receive pulses A and B.

The first process starts with pulse A at event (τ'_A, ξ') on the timelike hyperbolic world line $\xi' = const'$. in Rindler sector I. That pulse is launched from the instantaneous Lorentz frame $A(\tau'_A, \xi')$ centered around this event. Having traced out its world history across the future event horizon of I, this pulse ends the first process at event (ξ, τ_A) on the spacelike hyperbola of synchronous time $\xi = const$. There, in the local Lorentz frame $A(\xi, \tau_A)$ of the inertial recording clock with label $\tau = \tau_A$, the (mean) wavelength of the pulse is measured and recorded.

The second process starts with pulse B emitted at event (τ'_B, ξ') on the *same* timelike hyperbola $\xi' = const'$. but at *different* Rindler time $\tau' = \tau'_B$. This pulse also traces out a world history across the future event horizon. But the end of this pulse is at (ξ, τ_B) on the *same* spacelike hyperbola $\xi = const$. There the (mean) wavelength of the pulse gets measured relative the local Lorentz frame $B(\xi, \tau_B)$ of the inertial recording clock with *different* label $\tau = \tau_B$. Even though pulses A and B are emitted sequentially by one and the same transmitter, they are received simultaneously by two different recording clocks. This is made possible by the fact that the clock labelled by τ_B is moving towards the approaching pulse B. The blueshift resulting from this motion precisely compensates the redshift which pulse B has relative to A *if* the recording clock did not have this motion. Thus recording clocks τ_A and τ_B receive pulses A and B having precisely the same respective frequencies. This agreement is guaranteed by the principle of relativity. Indeed, Eq.(2) is a Lorentz transform of (3). Each consists of two events, two sets of frame vectors, and a straight pulse history. The Lorentz transformation maps these five entities associated with pulse A into those associated with pulse B:

$$
\begin{array}{ccccc}
(\tau'_A, \xi') : & A(\tau'_A, \xi') & \xrightarrow{\text{world history of pulse } A} & (\xi, \tau_A) : & A(\xi, \tau_A) \\
\downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\
(\tau'_B, \xi') : & B(\tau'_B, \xi') & \xrightarrow{\text{world history of pulse } B} & (\xi, \tau_B) : & B(\xi, \tau_B)
\end{array}
\tag{4}
$$

Thus the relative velocity, and hence the Doppler shift between frames $A(\xi', \tau'_A)$ and $A(\xi, \tau_A)$, is the same as that between $B(\tau'_B, \xi')$ and $B(\xi, \tau_B)$. This means that the wavelengths of the two received pulses at clock $A(\tau_A)$ and clock $B(\tau_B)$ are the same. There is *no Doppler chirp* in the composite spatial profile of the received e.m. field at fixed synchronous time ξ. If the emitted signal is monochromatic relative to the accelerated transmitter in I, then so is the spatial amplitude profile of the received signal relative to the expanding inertial frame in F. The transmission of a sequence of pulses is achieved with 100% fidelity.

This conclusion applies to all wavepackets. It also applies to any signal. This is because it is a linear superposition of such packets. A precise mathematical formulation of the emission of signals and their fidelity in transit from an accelerated source to an expanding inertial frame is developed in Section VI A.

Some authors thought that there is some sort of a disconnect between mathematics and physics, in particular between computations and what the computations refer to. For example, they claimed that "$\cdots$ the coordinates that we use [for computation] are arbitrary and have no physical meaning" [11] or "It is the very gist of relativity that anybody may use any frame [in his computations]." [12] Without delving into the epistemological fallacies underlying these claims, one should be aware of their unfortunate consequences. They tend to discourage attempts to understand natural processes whose very existence and identity one learns through measurements and computations based on nonarbitrary coordinate frames. The identification of radiation from violently accelerated bodies is a case in point. For these, two complementary frames are necessary: an accelerated frame to accommodate the source (Rindler sector I and/or II) and the corresponding expanding inertial frame (Rindler sector F) to observe the information carried by the radiation coming from this source. These frames are physically and geometrically distinct from static inertial frames. They also provide the logical connecting link between the concepts and the perceptual manifestations (measurements) of these radiation processes. Without these frames the concepts would not be concepts but mere floating abstractions.

IV. MAXWELL FIELDS: TRANSVERSE ELECTRIC AND TRANSVERSE MAGNETIC

The connecting link between the observed electromagnetic (e.m.) field and its source is the Maxwell field equations. The linear acceleration of the source, as well as the relative motion of the expanding set of recording clocks, determine the axis of a cylindrical geometry. This geometry results in the solutions to the e.m. field being decomposed into two distinct independently evolving fields the familiar T.M. polarized field and the T.E. field. Each is based on a single scalar field, which is a scalar under those Lorentz transformations which preserve cylindrical symmetry. In particular, the T.M. (resp. T.E.) field has vanishing magnetic (resp. electric) but nonzero electric (resp. magnetic) field parallel to the cylinder axis. Finally, there are also the T.E.M. fields. They are both T.E. and T.M. at the same time, and they propagate strictly parallel to the cylinder axis.

There is an analogous T.M.-T.E.-T.E.M. decomposition of the source. For example, the difference between the T.M. and the T.E. fields is that the source of the T.M. fields is the density of electric multipoles, while the source for the T.E. fields is the density of magnetic multipoles.

A. The Method of the 2+2 Split

The first task is to solve the inhomogeneous Maxwell field equations [13,14] and use its solution to determine the radiation properties to be measured. The best way to set up and solve these equations is to take advantage of the fact the cylindrical symmetry dictates a 2+2 decomposition of spacetime into a pair orthogonal 2-dimensional planes, one Euclidean, the other Lorentzian. The experienced reader will see that such a decomposition minimizes (compared to text book treatments) the amount of mathematical analysis, while simultaneously retaining all physical aspects of the radiation problem. Furthermore, these physical aspects lend themselves to nearly effortless identification because of the flexible curvilinear coordinate systems which the two orthogonal planes accommodate.

However, in order to appreciate the underlying line of reasoning as rapidly as possible, we first illustrate the 2+2 decomposition procedure on Minkowski spacetime coordinatized with the familiar rectilinear coordinates. We do this before proceeding to use it to solve Maxwell's equations relative to the various Rindler coordinates plus polar coordinates as called for by the cylindrical coordinate geometry of linearly accelerating bodies.

In the presence of cylindrical symmetry the Maxwell field equations decouple into two sets, each of which gives rise to its own inhomogeneous scalar wave equation

$$\left[\left(-\frac{\partial^2}{\partial t^2}+\frac{\partial^2}{\partial z^2}\right)+\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\right]\psi(t,z,x,y)=-4\pi\, S(t,z,x,y)\ . \tag{5}$$

The charge-current four-vector

$$S_\mu dx^\mu = S_t dt + S_z dz + S_x dx + S_y dy \tag{6}$$

for each set determines and is determined by appropriate derivatives of the scalar source $S(t,z,x,y)$. Similarly, the vector potential

$$A_\mu dx^\mu = A_t dt + A_z dz + A_x dx + A_y dy \tag{7}$$

as well as the electromagnetic field

$$\begin{aligned}\frac{1}{2}F_{\mu\nu}dx^\mu\wedge dx^\nu &\equiv \frac{1}{2}(\partial_\mu A_\nu-\partial_\nu A_\mu)dx^\mu\wedge dx^\nu\\ &\equiv \hat{E}_{long.}dz\wedge dt+\hat{E}_y dy\wedge dt+\hat{E}_x dx\wedge dt\\ &\quad+\hat{B}_x dy\wedge dz+\hat{B}_y dz\wedge dx+\hat{B}_{long.}dx\wedge dy\end{aligned}$$

determine and are determined by the scalar wave function $\psi(t,z,x,y)$ with the result that the Maxwell field equations

$$F_{\mu\ \ ;\nu}^{\ \nu}=4\pi S_\mu \tag{8}$$

are satisfied whenever the wave Eq.(5) is satisfied.

1. The T.E. Field

For the T.E. degrees of freedom the components of the charge-current four-vector are

$$(S_t,S_z,S_x,S_y)=\left(0,0,\frac{\partial S}{\partial y},-\frac{\partial S}{\partial x}\right)\ . \tag{9}$$

The components of the T.E. vector potential are

$$(A_t,A_z,A_x,A_y)=\left(0,0,\frac{\partial \psi}{\partial y},-\frac{\partial \psi}{\partial x}\right)\ , \tag{10}$$

and those of the e.m. field are

$E_{long.}$:	$F_{zt} = 0$
E_x :	$F_{xt} = -\dfrac{\partial}{\partial y}\dfrac{\partial \psi}{\partial t}$
E_y :	$F_{yt} = \dfrac{\partial}{\partial x}\dfrac{\partial \psi}{\partial t}$
$B_{long.}$:	$F_{xy} = -\left(\dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}\right)\psi$
B_x :	$F_{yz} = \dfrac{\partial}{\partial x}\dfrac{\partial \psi}{\partial z}$
B_y :	$F_{zx} = \dfrac{\partial}{\partial y}\dfrac{\partial \psi}{\partial z}$

These components are guaranteed to satisfy all the Maxwell field equations with T.E. source, Eq.(9), whenever ψ satisfies the inhomogeneous scalar wave equation, Eq.(5).

2. The T.M. Field

For the T.M. degrees of freedom the source and the electromagnetic field are also derived from a solution to the same inhomogeneous scalar wave Eq.(5). However, the difference from the T.E. case is that the four-vector components of the source and the vector potential lie in the Lorentz (t, z)-plane. Thus, instead of Eqs.(9) and (10), one has the T.M. source

$$(S_t, S_z, S_x, S_y) = \left(\frac{\partial S}{\partial z}, \frac{\partial S}{\partial t}, 0, 0\right) \tag{11}$$

and the T.M. vector potential

$$(A_t, A_z, A_x, A_y) = \left(\frac{\partial \psi}{\partial z}, \frac{\partial \psi}{\partial t}, 0, 0\right) \ . \tag{12}$$

All the corresponding T.M. field components are derived from the scalar $\psi(t, z, x, y)$:

$E_{long.}$:	$F_{zt} = \left(\dfrac{\partial^2}{\partial z^2} - \dfrac{\partial^2}{\partial t^2}\right)\psi$	
E_x :	$F_{xt} = \dfrac{\partial}{\partial x}\dfrac{\partial \psi}{\partial z}$	
E_y :	$F_{yt} = \dfrac{\partial}{\partial y}\dfrac{\partial \psi}{\partial z}$	
$B_{long.}$:	$F_{xy} = 0$	(13)
B_x :	$F_{yz} = \dfrac{\partial}{\partial y}\dfrac{\partial \psi}{\partial t}$	
B_y :	$F_{zx} = -\dfrac{\partial}{\partial x}\dfrac{\partial \psi}{\partial t}$	

These components are guaranteed to satisfy all the Maxwell field equations with the T.M. source, Eq.(11), whenever ψ satisfies the inhomogeneous scalar wave equation, Eq.(5).

3. The T.E.M. Field Equations

There are also the T.E.M. degrees of freedom. For them the Maxwell four-vector source

$$(S_t, S_z, S_x, S_y) = \left(\frac{\partial I}{\partial t}, \frac{\partial I}{\partial z}, \frac{\partial J}{\partial x}, \frac{\partial J}{\partial y}\right) \tag{14}$$

is derived from two functions $I(t,z,x,y)$ and, $J(t,z,x,y)$, scalars on the 2-D Lorentz plane and the 2-D-Euclidean plane respectively. They are, however, not independent. Charge conservation demands the relation

$$\left(\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial z^2}\right) I=\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right) J$$

The T.E.M. four-vector potential

$$(A_t,A_z,A_x,A_y)=\left(\frac{\partial\phi}{\partial t},\frac{\partial\phi}{\partial z},\frac{\partial\psi}{\partial x},\frac{\partial\psi}{\partial y}\right) \tag{15}$$

has the same form, but only the difference $\phi-\psi$ is determined by the field equations. Indeed, the T.E.M. field components are derived from this difference:

$E_{long.}$:	$F_{zt}=0$
E_x :	$F_{xt}=\partial_x A_t-\partial_t A_x=\dfrac{\partial}{\partial x}\dfrac{\partial(\phi-\psi)}{\partial t}$
E_y :	$F_{yt}=\partial_y A_t-\partial_t A_y=\dfrac{\partial}{\partial y}\dfrac{\partial(\phi-\psi)}{\partial t}$
$B_{long.}$:	$F_{xy}=0$
B_x :	$F_{yz}=\partial_y A_z-\partial_z A_y=\dfrac{\partial}{\partial y}\dfrac{\partial(\phi-\psi)}{\partial z}$
B_y :	$F_{zx}=\partial_z A_x-\partial_x A_z=-\dfrac{\partial}{\partial x}\dfrac{\partial(\phi-\psi)}{\partial z}$

This e.m. field satisfies the Maxwell field equations if any two of the following three scalar equations,

$$-\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)(\phi-\psi)=4\pi I \tag{16}$$

$$\left(-\frac{\partial^2}{\partial t^2}+\frac{\partial^2}{\partial z^2}\right)(\phi-\psi)=4\pi J \tag{17}$$

$$\left(\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial z^2}\right) I=\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right) J \tag{18}$$

are satisfied. The last equation is, of course, simply the conservation of charge equation. Furthermore, it is evident that the T.E.M. field propagates strictly along the z-axis, the direction of the Pointing vector.

B. Existence and Uniqueness of the Method of the 2+2 Split

Can an arbitrary vector potential be written in terms of four suitably chosen scalars $\psi^{T.E.},\psi^{T.M.},\psi$ and ϕ so as to satisfy Eqs.(10), (12), and (15)? If the answer is "yes" then any e.m. field can be expressed in terms of these scalars, and one can claim that these scalars give a complete and equivalent description of the e.m. field. It turns out that this is indeed the case. In fact, the description is also unique. Indeed, given the vector potential, Eq.(7), there exist four unique scalars which are determined by this vector potential so as to satisfy Eqs.(10), (12), and (15). The determining equations, obtained by taking suitable derivatives, are

$$\frac{\partial^2\psi^{T.E.}}{\partial x^2}+\frac{\partial^2\psi^{T.E.}}{\partial y^2}=\partial_y A_x-\partial_x A_y \tag{19}$$

$$-\frac{\partial^2\psi^{T.M.}}{\partial t^2}+\frac{\partial^2\psi^{T.M.}}{\partial z^2}=\partial_z A_t-\partial_t A_z \tag{20}$$

$$\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}=\partial_x A_x+\partial_y A_y \tag{21}$$

$$-\frac{\partial^2\phi}{\partial t^2}+\frac{\partial^2\phi}{\partial z^2}=-\partial_t A_t+\partial_z A_z \tag{22}$$

These equations guarantee the existence of the sought after scalar functions $\psi^{T.E.}, \psi^{T.M.}, \psi$ and ϕ . Their uniqueness follows from their boundary conditions in the Euclidean (x, y)-plane and their initial conditions in the Lorentzian (t, z)-plane. Consequently, Eqs.(19)-(22) together with Eqs.(10), (12), and (15) establish a one-to-one linear correspondence between the space of vector potentials and the space of four ordered scalars,

$$(\psi^{T.E.}, \psi^{T.M.}, \psi, \phi) \longleftrightarrow (A_t, A_z, A_x, A_y)$$

Of the four scalars, three are gauge invariants, namely $\psi^{T.E.}$, $\psi^{T.M.}$, and the difference $\psi - \phi$, a result made obvious by inspecting Eqs.(19)-(22).

C. Historical Remarks

The T.E. scalar and the T.M. scalar whose derivatives yield the respective vector potentials Eqs.(10) and (12) can be related to Righi's magnetic "super potential" vector $\vec{\Pi}^m$ and Hertz's electric "super potential" vector $\vec{\Pi}^e$ [15]. Indeed, if $\psi_{T.E.}$ is the T.E. scalar and $\psi_{T.M.}$ is the T.M. scalar, then these scalars are simply the z-components of the corresponding super potential vectors

$$\vec{\Pi}^m \equiv (\Pi^m_x, \Pi^m_y, \Pi^m_z) = (0, 0, \psi_{T.E.}) \quad \text{"Righi"} \tag{23}$$
$$\vec{\Pi}^e \equiv (\Pi^e_x, \Pi^e_y, \Pi^e_z) = (0, 0, \psi_{T.M.}) \quad \text{"Hertz"} \tag{24}$$

In fact, subsequent to Hertz's 1900 and Righi's 1901 introduction of their super potential vectors, Whittaker in 1903 showed that the Maxwell field can be derived precisely from our two gauge invariant scalars ψ^{TE} and ψ^{TM} [15].

D. Application to Accelerated and Expanding Inertial Frames

A key virtue of splitting spacetime according to the 2+2 scheme is its flexibility. It accommodates the necessary Rindler coordinate geometries which are called for by the physical problem: accelerated frames for the accelerated sources, and expanding inertial frames for the inertial observers who measure the radiation emitted from these sources. These geometries are

$$ds^2 = -\xi^2 d\tau^2 + d\xi^2 + dr^2 + r^2 d\theta^2 \qquad \text{in } I \text{ or in } II \quad (\text{"accelerated frame"}) \tag{25}$$

and

$$ds^2 = -d\xi^2 + \xi^2 d\tau^2 + dr^2 + r^2 d\theta^2 \qquad \text{in } F \text{ or in } P \quad (\text{"expanding (or contracting) inertial frame"}) \tag{26}$$

In these two frames the Rindler/polar-coordinatized version of Eq.(5) is

$$\left[\left(-\frac{1}{\xi^2}\frac{\partial^2}{\partial\tau} + \frac{1}{\xi}\frac{\partial}{\partial\xi}\xi\frac{\partial}{\partial\xi}\right) + \left(\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\theta}\right)\right]\psi(\tau,\xi,r,\theta) = -4\pi S(\tau,\xi,r,\theta) \qquad \text{in } I \text{ or in } II \ , \tag{27}$$

and

$$\left[\left(-\frac{1}{\xi}\frac{\partial}{\partial\xi}\xi\frac{\partial}{\partial\xi} + \frac{1}{\xi^2}\frac{\partial^2}{\partial\tau}\right) + \left(\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\theta}\right)\right]\psi(\xi,\tau,r,\theta) = 0 \qquad \text{in } F \text{ or in } P \ . \tag{28}$$

Notational rule: The Rindler coordinates listed in the arguments of the scalar functions in Eqs.(27) and (28) are always listed with the timelike coordinate first, followed by the spatial coordinates. Thus (τ, ξ, r, θ) implies that the function is defined on Rindler sectors I or II, as in Eq.(27). On the other hand, (ξ, τ, r, θ) implies that the domain of the function is F or P, as in Eq.(28).

The feature common to the T.E. and the T.M. field is that both of them are based on the two-dimensional curl of a scalar, say ψ. The difference is that for the T.E. field this curl is in the Euclidean plane,

$$\nabla_a \times \psi \equiv \epsilon_{ab}\, {}^{(2)}g^{bc}\frac{\partial\psi}{\partial x^c} : \quad (\nabla_r \times \psi, \nabla_\theta \times \psi) = \left(\frac{1}{r}\frac{\partial\psi}{\partial\theta}, -r\frac{\partial\psi}{\partial r}\right) \ ,$$

while for the T.M. field this curl is the Lorentz plane,

$$\nabla_A \times \psi \equiv \epsilon_{AB}\ {}^{(2)}g^{BC}\frac{\partial \psi}{\partial x^C}: \quad (\nabla_\tau \times \psi, \nabla_\xi \times \psi) = \left(\xi\frac{\partial \psi}{\partial \xi}, \frac{1}{\xi}\frac{\partial \psi}{\partial \tau}\right) \quad \text{in } I \text{ or in } II \ ,$$

and

$$\nabla_A \times \psi \equiv \epsilon_{AB}\ {}^{(2)}g^{BC}\frac{\partial \psi}{\partial x^C}: \quad (\nabla_\xi \times \psi, \nabla_\tau \times \psi) = \left(\frac{1}{\xi}\frac{\partial \psi}{\partial \tau}, \xi\frac{\partial \psi}{\partial \xi}\right) \quad \text{in } F \text{ or in } P \ ,$$

The ϵ_{ab} and ϵ_{AB} are the components of the antisymmetric area tensors on the two respective planes.

1. The T.E. Field

For the T.E. degrees of freedom the charge-current $S_\mu dx^\mu$ and the vector potential $A_\mu dx^\mu$ have the form given by

$$(S_\tau, S_\xi, S_r, S_\theta) = \left(0, 0, \frac{1}{r}\frac{\partial S}{\partial \theta}, -r\frac{\partial S}{\partial r}\right) \ , \tag{29}$$

and

$$(A_\tau, A_\xi, A_r, A_\theta) = \left(0, 0, \frac{1}{r}\frac{\partial \psi}{\partial \theta}, -r\frac{\partial \psi}{\partial r}\right) \ . \tag{30}$$

The electromagnetic field,

$$\begin{aligned}\frac{1}{2}F_{\mu\nu}dx^\mu \wedge dx^\nu \equiv\ & \hat{E}_{long.}d\xi \wedge \xi d\tau + \hat{E}_r dr \wedge \xi d\tau + \hat{E}_\theta r d\theta \wedge \xi d\tau \\ & + \hat{B}_r r d\theta \wedge d\xi + \hat{B}_\theta d\xi \wedge dr + \hat{B}_{long.} dr \wedge r d\theta \qquad \text{in } I \text{ and } II \\ \equiv\ & \hat{E}_{long.}\xi d\tau \wedge d\xi + \hat{E}_r dr \wedge d\xi + \hat{E}_\theta r d\theta \wedge d\xi \\ & + \hat{B}_r r d\theta \wedge \xi d\tau + \hat{B}_\theta \xi d\tau \wedge dr + \hat{B}_{long.} dr \wedge r d\theta \qquad \text{in } P \text{ and } F \ ,\end{aligned}$$

has the following components:

	In I or in II	In F or in P
$\hat{E}_{long.}$:	$\frac{1}{\xi}F_{\xi\tau} = 0$	$\frac{1}{\xi}F_{\tau\xi} = 0$
$\hat{E}_r$:	$\frac{1}{\xi}F_{r\tau} = -\frac{1}{r}\frac{\partial}{\partial \theta}\left(\frac{1}{\xi}\frac{\partial \psi}{\partial \tau}\right)$	$F_{r\xi} = -\frac{1}{r}\frac{\partial}{\partial \theta}\left(\frac{\partial \psi}{\partial \xi}\right)$
$\hat{E}_\theta$:	$\frac{1}{\xi r}F_{\theta\tau} = \frac{\partial}{\partial r}\left(\frac{1}{\xi}\frac{\partial \psi}{\partial \tau}\right)$	$\frac{1}{r}F_{\theta\xi} = \frac{\partial}{\partial r}\left(\frac{\partial \psi}{\partial \xi}\right)$
$\hat{B}_{long.}$:	$\frac{1}{r}F_{r\theta} = -\left(\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}\right)\psi$	$\frac{1}{r}F_{r\theta} = -\left(\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}\right)\psi$
$\hat{B}_r$:	$\frac{1}{r}F_{\theta\xi} = \frac{\partial}{\partial r}\left(\frac{\partial \psi}{\partial \xi}\right)$	$\frac{1}{\xi r}F_{\theta\tau} = \frac{\partial}{\partial r}\left(\frac{1}{\xi}\frac{\partial \psi}{\partial \tau}\right)$
$\hat{B}_\theta$:	$F_{\xi r} = \frac{1}{r}\frac{\partial}{\partial \theta}\left(\frac{\partial \psi}{\partial \xi}\right)$	$\frac{1}{\xi}F_{\tau r} = \frac{1}{r}\frac{\partial}{\partial \theta}\left(\frac{1}{\xi}\frac{\partial \psi}{\partial \tau}\right)$

The carets in the first column serve as a reminder that these components are relative to the orthonormal basis of the metric, Eqs.(25) and (26).

2. The T.M. Field

The T.M. has its source and vector potential four-vectors lie strictly in the 2-d Lorentz plane:

$$(S_\tau, S_\xi, S_r, S_\theta) = \left(\xi\frac{\partial S}{\partial \xi}, \frac{1}{\xi}\frac{\partial S}{\partial \tau}, 0, 0\right) \quad \text{in } I \text{ or in } II \ , \tag{31}$$

$$(A_\tau, A_\xi, A_r, A_\theta) = \left(\xi\frac{\partial\psi}{\partial\xi}, \frac{1}{\xi}\frac{\partial\psi}{\partial\tau}, 0, 0\right) \quad \text{in } I \text{ or in } II \ , \tag{32}$$

and

$$(A_\xi, A_\tau, A_r, A_\theta) = \left(\frac{1}{\xi}\frac{\partial\psi}{\partial\tau}, \xi\frac{\partial\psi}{\partial\xi}, 0, 0\right) \quad \text{in } F \text{ or in } P \tag{33}$$

The components of the T.M. Maxwell field are

	In I or in II	In F or in P
$\hat{E}_{long.}$:	$\frac{1}{\xi}F_{\xi\tau} = \left(\frac{1}{\xi}\frac{\partial}{\partial\xi}\xi\frac{\partial}{\partial\xi} - \frac{1}{\xi^2}\frac{\partial^2}{\partial\tau^2}\right)\psi$	$\frac{1}{\xi}F_{\tau\xi} = -\left(\frac{1}{\xi}\frac{\partial}{\partial\xi}\xi\frac{\partial}{\partial\xi} - \frac{1}{\xi^2}\frac{\partial^2}{\partial\tau^2}\right)\psi$
$\hat{E}_r$:	$\frac{1}{\xi}F_{r\tau} = \frac{\partial}{\partial r}\left(\frac{\partial\psi}{\partial\xi}\right)$	$F_{r\xi} = \frac{\partial}{\partial r}\left(\frac{1}{\xi}\frac{\partial\psi}{\partial\tau}\right)$
$\hat{E}_\theta$:	$\frac{1}{\xi r}F_{\theta\tau} = \frac{1}{r}\frac{\partial}{\partial\theta}\left(\frac{\partial\psi}{\partial\xi}\right)$	$\frac{1}{r}F_{\theta\xi} = \frac{1}{r}\frac{\partial}{\partial\theta}\left(\frac{1}{\xi}\frac{\partial\psi}{\partial\tau}\right)$
$\hat{B}_{long.}$:	$\frac{1}{r}F_{r\theta} = 0$	$\frac{1}{r}F_{r\theta} = 0$
$\hat{B}_r$:	$\frac{1}{r}F_{\theta\xi} = \frac{1}{r}\frac{\partial}{\partial\theta}\left(\frac{1}{\xi}\frac{\partial\psi}{\partial\tau}\right)$	$\frac{1}{\xi r}F_{\theta\tau} = \frac{1}{r}\frac{\partial}{\partial\theta}\left(\frac{\partial\psi}{\partial\xi}\right)$
$\hat{B}_\theta$:	$F_{\xi r} = -\frac{\partial}{\partial r}\left(\frac{1}{\xi}\frac{\partial\psi}{\partial\tau}\right)$	$\frac{1}{\xi}F_{\tau r} = -\frac{\partial}{\partial r}\left(\frac{\partial\psi}{\partial\xi}\right)$

3. A Mnemonic Short Cut

There is a quick way of obtaining all the physical (orthonormal) components of the electric and magnetic field. Note that the longitudinal electric and magnetic field components $\hat{E}_{long}$ and $\hat{B}_{long}$ are scalars in the Lorentz plane and in the Euclidean plane transverse to it. Consequently, for these components the transition from Minkowski to Rindler/polar coordinates could have been done without any computations. The same is true for the two-dimensional transverse electric and magnetic field vectors. As suggested by Eqs.(25) and (26), in the denominator of the partial derivatives simply make the replacements

$$\begin{aligned} \partial t &\to \xi\partial\tau \\ \partial z &\to \partial\xi \\ \partial x &\to \partial r \\ \partial y &\to r\partial\theta \end{aligned}$$

in Rindler sectors I or II, and

$$\begin{aligned} \partial t &\to \partial\xi \\ \partial z &\to \xi\partial\tau \\ \partial x &\to \partial r \\ \partial y &\to r\partial\theta \end{aligned}$$

in Rindler sectors F or P. These replacements yield the computed transverse T.E. and T.M. components.

There also is a quick way of obtaining the T.M. field from the T.E. field components. Let ψ^{TE} be the scalar wave function which satisfies the Klein-Gordon wave function for the T.E. field, and let ψ^{TM} be that for the T.M. field. Then the corresponding field components are related as follows:

$$\begin{aligned} &T.E. \qquad T.M. \\ &\psi^{TE} \to \quad \psi^{TM} \\ \hat{E}_{long} &= 0 \to -\hat{B}_{long} = 0 \\ \hat{E}_r &\to -\hat{B}_r \end{aligned}$$

$$\begin{array}{rl} \hat{E}_\theta \to & -\hat{B}_\theta \\ \hat{B}_{long} \to & \hat{E}_{long} \qquad (in\ vacuum) \\ \hat{B}_r \to & \hat{E}_r \\ \hat{B}_\theta \to & \hat{E}_\theta \end{array}$$

This relationship holds in all four Rindler sectors. It also holds correspondingly relative to the rectilinear coordinate frame in Sections IV A 2 and IV A 1.

V. RADIATION: MATHEMATICAL RELATION TO THE SOURCE

Any Maxwell field $F_{\mu\nu}$ can be obtained from a single Klein-Gordon scalar field ψ, a solution to the scalar wave Eq.(5). This is done with the help of the T.E. and T.M. tables of derivatives. Similarly, any K-G field ψ can be obtained from the source function S. This is done with the help of the unit impulse response $\mathcal{G}$ (Green's function), the solution to

$$\left(-\frac{\partial^2}{\partial t^2}+\frac{\partial^2}{\partial z^2}+\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\theta}\ .\right)\mathcal{G}=-\delta(t-t')\delta(z-z')\frac{\delta(r-r')}{r}\delta(\theta-\theta') \tag{34}$$

In terms of $\mathcal{G}$ the solution to the inhomogeneous wave equation, Eq.(27) is

$$\psi(t,z,r,\theta)=\int_{-\infty}^{\infty}\int_0^{\infty}\int_0^{\infty}\int_0^{2\pi}\mathcal{G}(t,z,r,\theta;\tau',\xi',r',\theta')\ 4\pi S(t',z',r',\theta')\ \ dt'\,dz'\,r'dr'\,d\theta'\ , \tag{35}$$

Here S is the scalar source, which is non-zero only in Rindler sectors I and II.

A. Unit Impulse Response

The solution to Eq.(34) is the retarded Green's function, a unique scalar field, whose domain extends over all four Rindler sectors. One accommodates the cylindrical symmetry of the coordinate geometry by representing the scalar field in terms of the appropriate eigenfunctions, the Bessel harmonics $J_m(kr)e^{im\theta}$, for the Euclidean (r,θ)-plane:

$$\mathcal{G}(t,z,r,\theta;t',z',r',\theta')=\sum_{m=-\infty}^{\infty}\int_0^{\infty}G(t,z;t',z')\ J_m(kr)\frac{e^{im(\theta-\theta')}}{2\pi}J_m(kr')\ k\,dk\ , \tag{36}$$

where G satisfies

$$\left(-\frac{\partial^2}{\partial t^2}+\frac{\partial^2}{\partial z^2}-k^2\right)G=-\delta(t-t')\delta(z-z')\ , \tag{37}$$

and

$$G=0\quad\text{whenever } t<t'\ .$$

This Green's function is unique, and it is easy to show [16] that

$$G=\begin{cases}\frac{1}{2}J_0(k\sqrt{(t-t')^2-(z-z')^2}) & \text{whenever } t-t'\geq|z-z'|\\ 0 & \text{whenever } t-t'<|z-z'|\ .\end{cases} \tag{38}$$

This means that G is non-zero only inside the future of the source event (t',z'), and vanishes identically everywhere else. The function $G(t,z;t',z')$ is defined on all four Rindler sectors. However, our interest is only in those of its coordinate representatives whose source events lie Rindler sectors I or II,

$$\left.\begin{array}{ll} t' & =\pm\xi'\sinh\tau' \\ z' & =\pm\xi'\cosh\tau' \end{array}\right\}\left\{\begin{array}{l}\text{upper sign for } I\\ \text{lower sign for } II\ ,\end{array}\right.$$

and whose observation events lie in Rindler sector F,

$$
\begin{aligned}
t &= \xi \cosh \tau \\
z &= \xi \sinh \tau \ .
\end{aligned}
$$

For these coordinate restrictions the two coordinate representatives of $G(t,z;t',z')$, Eq.(38), are

$$
G_I(k\xi,\tau;\tau',k\xi') = \begin{cases} \frac{1}{2} J_0 \left(k\sqrt{\xi^2 - \xi'^2 + 2\xi\xi' \sinh(\tau - \tau')} \right) & \text{whenever } (\xi,\tau) \text{ is in } F \text{ and } (\xi',\tau') \text{ is in } I \\ 0 & \text{whenever } \xi^2 - \xi'^2 + 2\xi\xi' \sinh(\tau-\tau') < 0 \end{cases} \tag{39}
$$

and

$$
G_{II}(k\xi,\tau;\tau',k\xi') = \begin{cases} \frac{1}{2} J_0 \left(k\sqrt{\xi^2 - \xi'^2 - 2\xi\xi' \sinh(\tau - \tau')} \right) & \text{whenever } (\xi,\tau) \text{ is in } F \text{ and } (\xi',\tau') \text{ is in } II \\ 0 & \text{whenever } \xi^2 - \xi'^2 - 2\xi\xi' \sinh(\tau-\tau') < 0 \end{cases} \tag{40}
$$

These two coordinate representatives give rise to the corresponding two representatives of the unit impulse response, Eq.(36),

$$
\mathcal{G}_{I,II}(\xi,\tau,r,\theta;\tau',\xi',r',\theta') = \int_0^\infty G_{I,II}(k\xi,\tau;\tau',k\xi') \sum_{m=-\infty}^{\infty} J_m(kr) \frac{e^{im(\theta-\theta')}}{2\pi} J_m(kr') \; k\, dk \ , \tag{41}
$$

This integral expression is exactly what is needed to obtain the radiation field from bodies accelerated in I and/or II. However, in order to ascertain agreement with previously established knowledge, we shall use the remainder of this subsection to evaluate the sum and the integral in Eq.(41) explicitly.

It is a delightful property of Bessel harmonics that the sum over m can be evaluated in closed form [17]. This property is the Euclidean plane analogue of what for spherical harmonics is the spherical addition theorem. One has

$$
\sum_{m=-\infty}^{\infty} J_m(kr) \frac{e^{im(\theta-\theta')}}{2\pi} J_m(kr') = \frac{1}{2\pi} J_0 \left(k\sqrt{r^2 + r'^2 - 2rr' \cos(\theta - \theta')} \right) \tag{42}
$$

Inserting this result, as well as Eqs.(39) or (40) into Eq.(41) yields the two unit impulse response functions with sources in I (upper sign) and II (lower sign)

$$
\begin{aligned}
\mathcal{G}_{I,II}(\xi,\tau,r,\theta;\tau',\xi',r',\theta') &= \frac{1}{4\pi} \int_0^\infty J_0 \left(k\sqrt{\xi^2 - \xi'^2 \pm 2\xi\xi' \sinh(\tau-\tau')} \right) J_0 \left(k\sqrt{r^2 + r'^2 - 2rr' \cos(\theta-\theta')} \right) k\, dk \\
&= \frac{1}{4\pi} \int_0^\infty J_0 \left(k\sqrt{(t-t')^2 - (z-z')^2} \right) J_0 \left(k\sqrt{(x-x')^2 - (y-y')^2} \right) \; k\, dk
\end{aligned}
$$

whenever $t - t' \geq |z - z'|$ and zero otherwise. The spread-out amplitudes of this linear superposition interfere constructively to form a Dirac delta function response. Indeed, using the standard representation

$$
\int_0^\infty J_0(ka) J_0(kb) \;\, kdk = \frac{\delta(a-b)}{b}
$$

for this function, one finds that

$$
\begin{aligned}
\mathcal{G}_{I,II}(\xi,\tau,r,\theta;\tau',\xi',r',\theta') &= \frac{1}{4\pi} \frac{\delta \left(\sqrt{\xi^2 - \xi'^2 \pm 2\xi\xi' \sinh(\tau-\tau')} - \sqrt{r^2 + r'^2 - 2rr' \cos(\theta-\theta')} \right)}{\sqrt{r^2 + r'^2 - 2rr' \cos(\theta-\theta')}} \\
&= \frac{1}{2\pi} \delta \left(\xi^2 - \xi'^2 \pm 2\xi\xi' \sinh(\tau-\tau') - (r^2 + r'^2 - 2rr' \cos(\theta-\theta')) \right) \\
&= \frac{1}{2\pi} \delta \left((t-t')^2 - (z-z')^2 - (x-x')^2 - (y-y')^2 \right)
\end{aligned} \tag{43}
$$

whenever (t,z,x,y) is in the future of (t',z',x',y'). This is the familiar causal response in F due to a unit impulse event in I or in II.

B. Full Scalar Radiation Field

The scalar field measured in Rindler sector F of the expanding inertial reference frame is the linear superposition

$$\psi_F(\xi,\tau,r,\theta)=\psi_I(\xi,\tau,r,\theta)+\psi_{II}(\xi,\tau,r,\theta) \tag{44}$$

of two contributions. They arise from two causally disjoint sources, one in Rindler sector I, the other in Rindler sector II. Applying Eq.(41) together with Eqs.(39) and (40) to Eq.(35), one finds that these contributions are

$$\psi_{I,II}(\xi,\tau,r,\theta)=\sum_{m=-\infty}^{\infty}\int_0^\infty k\,dk e^{im\theta}J_m(kr)\times$$
$$\int_{-\infty}^{\infty}d\tau'\int_0^\infty \xi' d\xi'\frac{1}{2}J_0\left(k\sqrt{\xi^2-\xi'^2\pm 2\xi\xi'\sinh(\tau-\tau')}\right)\int_0^\infty r'dr'\int_0^{2\pi}d\theta'\frac{e^{-im\theta'}}{2\pi}J_m(kr')\;4\pi S_{I,II}(\tau',\xi',r',\theta') \tag{45}$$

These radiation fields are exact in the sense that we have made no assumptions about the size of the sources in relation to their radiated wavelengths. The only restriction is that the two source functions S_I and S_{II} be non-zero only in I and II respectively.

The key to identifying the nature of these fields is the evaluation of the mode sum over m and the mode integral over k. These evaluations have been done already starting with Eq.(42), and gave rise to the familiar causal unit impulse response, Eq.(43). However, for the purpose of physical transparency we shall not make these evaluations. Instead, we shall make an evaluation which is based on the assumption that each source is small compared to its radiated wave lengths k^{-1}. Physically this means that the phase of the e.m. field is the same across the whole extent of the source. Mathematically this means that the Bessel function $J_0(kr')$ should be replaced by the expression Eq.(46) below. The advantage of the long wave length approximation is physical/technological: it permits the characterization of the source in terms of multipole moments, which are directly tied to the angular distribution of the emitted radiation, and which derive their importance from, among others, the quantum mechanical selection rules.

1. Source as a Sum of Multipoles

The circumstance of long wave lengths is expressed by the inequality

$$ka \ll 1\ ,$$

where a is the radius of the cylinder surrounding the source. This circumstance allows us to set

$$J_m(kr')\approx\begin{cases}\dfrac{1}{m!}\left(\dfrac{kr'}{2}\right)^m & m=0,1,2,\cdots\\[2ex] \dfrac{(-1)^m}{|m|!}\left(\dfrac{kr'}{2}\right)^{|m|} & m=0,-1,-2,\cdots\end{cases} \tag{46}$$

throughout the integration region where the source is non-zero, and it allows us to introduce the $(m+1)$st multipole moment (per unit length $d\xi$)

$$\frac{i^m}{|m|!}\int_0^\infty r'dr'\int_0^{2\pi}d\theta'\frac{e^{-im\theta'}}{2\pi}\left(\frac{r'}{2}\right)^{|m|}\;4\pi S_{I,II}(\tau',\xi',r',\theta')\equiv 2S^m_{I,II}(\tau',\xi')\quad\left[\frac{\text{charge}}{\text{length}}\times(\text{length})^{|m|+1}\right] \tag{47}$$

for the double integral on the right hand side of Eq.(45). This multipole density [18] is complex. However, the reality of the master source $S_{I,II}(\tau',\xi',r',\theta')$ implies and is implied by

$$\overline{S^m_{I,II}(\tau',\xi')}=S^{-m}_{I,II}(\tau',\xi')$$

In terms of this multipole density the full scalar radiation field in F is

$$\psi_F(\xi,\tau,r,\theta)=\sum_{m=-\infty}^{\infty}\int_0^\infty dk\,k\,k^{|m|}e^{im\theta}J_m(kr)\times$$
$$\int_{-\infty}^{\infty}d\tau'\int_0^\infty d\xi'\xi'\,\frac{2}{2}\left\{J_0\left(k\sqrt{\xi^2-\xi'^2+2\xi\xi'\sinh(\tau-\tau')}\right)2S^m_I(\tau',\xi')+J_0\left(k\sqrt{\xi^2-\xi'^2-2\xi\xi'\sinh(\tau-\tau')}\right)2S^m_{II}(\tau',\xi')\right\} \tag{48}$$

The evaluation of the mode integral $\int_0^\infty dk\, k \cdots$ is now an easy two step task. First recall the mth recursion relation

$$e^{im\theta} J_m(kr) = \frac{(-1)^m}{k^{|m|}} \left[e^{i\theta} \left(\frac{\partial}{\partial r} + \frac{i}{r} \frac{\partial}{\partial \theta} \right) \right]^m J_0(kr), \quad m = 0, \pm 1, \pm 2, \cdots \tag{49}$$

where for negative m one uses

$$\left[e^{i\theta} \left(\frac{\partial}{\partial r} + \frac{i}{r} \frac{\partial}{\partial \theta} \right) \right]^{-|m|} \equiv \left[-e^{-i\theta} \left(\frac{\partial}{\partial r} - \frac{i}{r} \frac{\partial}{\partial \theta} \right) \right]^{|m|} .$$

This recursion relation is a consequence of consolidating two familiar contiguity relations for the Bessel functions. Introduce Eq.(49) into the integrand of Eq.(48).

Second, use the standard expression

$$\int_0^\infty J_0(kr) J_0(k\sqrt{\cdots}) k\, dk = \frac{\delta(r - \sqrt{\cdots})}{\sqrt{\cdots}}$$

for the Dirac delta function. Apply this equation to Eq.(48). Consequently, the full scalar radiation field in Rindler sector F reduces to the following multipole expansion

$$\psi_F(\xi, \tau, r, \theta) = \sum_{m=-\infty}^{\infty} (-1)^m \left[e^{i\theta} \left(\frac{\partial}{\partial r} + \frac{i}{r} \frac{\partial}{\partial \theta} \right) \right]^m \psi_m(\xi, \tau, r) \qquad \text{[charge]} \tag{50}$$

where

$$\psi_m(\xi,\tau,r) = \int_{-\infty}^{\infty} \int_0^\infty \frac{1}{2} \left\{ \frac{2S_I^m(\tau',\xi')}{\sqrt{\xi^2 - \xi'^2 + 2\xi\xi' \sinh(\tau - \tau')}} \delta\left(r - \sqrt{\xi^2 - \xi'^2 + 2\xi\xi' \sinh(\tau - \tau')} \right) + \right.$$
$$\left. \frac{2S_{II}^m(\tau',\xi')}{\sqrt{\xi^2 - \xi'^2 - 2\xi\xi' \sinh(\tau - \tau')}} \delta\left(r - \sqrt{\xi^2 - \xi'^2 - 2\xi\xi' \sinh(\tau - \tau')} \right) \right\} d\tau' \xi'\, d\xi' .$$

Doing the τ'-integration yields

$$\psi_m(\xi,\tau,r) = 2 \int_0^\infty \frac{[S_I^m(\tau',\xi')]_I - [S_{II}^m(\tau',\xi')]_{II}}{\sqrt{(\xi^2 - \xi'^2 - r^2)^2 + (2\xi\xi')^2}} \xi' d\xi' . \tag{51}$$

Here $[\ \ ]_I$ and $[\ \ ]_{II}$ mean that the source functions are evaluated in compliance with the Dirac delta functions at $\tau' = \tau + \sinh^{-1} \frac{\xi^2 - \xi'^2 - r^2}{2\xi\xi'}$ and $\tau' = \tau - \sinh^{-1} \frac{\xi^2 - \xi'^2 - r^2}{2\xi\xi'}$ respectively. Recall that τ' is a strictly timelike coordinate in Rindler sector I, while in F the coordinate τ is strictly spacelike. Consequently, one should not be tempted to identify $[\ \ ]_I$ and $[\ \ ]_{II}$ with what in a static inertial frame corresponds to evaluations at advanced or retarded times. Instead, one should think of the observation event (ξ, τ, r) in F and the source event (τ', ξ', r) in I as lying on each other's light cones

$$(t - t')^2 - (z - z')^2 = r^2 ,$$

both of which cut across the future event horizons $t = |z|$ of I and II. More explicitly, one has

$$[S_I^m(\tau',\xi')]_I \equiv S_I^m \left(\tau + \sinh^{-1} \frac{\xi^2 - \xi'^2 - r^2}{2\xi\xi'}, \xi' \right) ,$$

which means that the source $S_I^m(\tau',\xi')$ has been evaluated on the past light cone

$$(t - t')^2 - (z - z')^2 \equiv \xi^2 - \xi'^2 + 2\xi\xi' \sinh(\tau - \tau') = r^2$$

of (ξ, τ, r) at $(\tau', \xi', 0)$ in Rindler sector I. Similarly,

$$[S_{II}^m(\tau',\xi')]_{II} \equiv S_{II}^m \left(\tau - \sinh^{-1} \frac{\xi^2 - \xi'^2 - r^2}{2\xi\xi'}, \xi' \right) ,$$

which means that the source $S_{II}^m(\tau',\xi')$ has been evaluated on the past light cone

$$(t - t')^2 - (z - z')^2 \equiv \xi^2 - \xi'^2 - 2\xi\xi' \sinh(\tau - \tau') = r^2$$

of (ξ, τ, r) at $(\tau', \xi', 0)$ in Rindler sector II. The expression, Eq.(51), for the full scalar radiation field is exact within the context of wavelengths large compared to the size of the source. Furthermore, one should note that even though there is only one ξ'-integral, S_I^m and S_{II}^m are source functions with distinct domains, namely, Rindler sectors I and II respectively.

2. Multipole Radiation Field

The field is a superposition of multipole field amplitudes. The first few terms of this superposition are

$$
\begin{aligned}
\psi_F(\xi,\tau,r,\theta) &= \psi_0(\xi,\tau,r) \\
&- e^{i\theta}\frac{\partial}{\partial r}\psi_1(\xi,\tau,r) \\
&+ e^{2i\theta}\left(\frac{\partial^2}{\partial r^2}-\frac{1}{r}\frac{\partial}{\partial r}\right)\psi_2(\xi,\tau,r) \\
&- e^{3i\theta}\left(\frac{\partial^3}{\partial r^3}-\frac{3}{r}\frac{\partial^2}{\partial r^2}+\frac{3}{r^2}\frac{\partial}{\partial r}\right)\psi_3(\xi,\tau,r) \\
&+ e^{4i\theta}\left(\frac{\partial}{\partial r^4}-\frac{6}{r}\frac{\partial^3}{\partial r^3}+\frac{12}{r^2}\frac{\partial^2}{\partial r^2}-\frac{15}{r^3}\frac{\partial}{\partial r}\right)\psi_4(\xi,\tau,r) \ + \quad \cdots \\
&+ (\text{complex conjugate terms corresponding to } m=-1,-2,-3,\cdots)
\end{aligned}
\tag{52}
$$

whose explicit form is

$$
\begin{aligned}
\psi_F(\xi,\tau,r,\theta) &= \int_0^\infty \left[\frac{2S_I^0(\tau+\sinh^{-1}u,\xi')-2S_{II}^0(\tau-\sinh^{-1}u,\xi')}{\sqrt{(\xi^2-\xi'^2-r^2)^2+(2\xi\xi')^2}}\right]\xi'\,d\xi' \\
&- e^{i\theta}\frac{\partial}{\partial r}\left[\frac{2S_I^1(\tau+\sinh^{-1}u,\xi')-2S_{II}^1(\tau-\sinh^{-1}u)}{\sqrt{(\xi^2-\xi'^2-r^2)^2+(2\xi\xi')^2}}\right]\xi'\,d\xi' \\
&+ e^{2i\theta}\left(\frac{\partial^2}{\partial r^2}-\frac{1}{r}\frac{\partial}{\partial r}\right)\left[\frac{2S_I^2(\tau+\sinh^{-1}u,\xi')-2S_{II}^2(\tau-\sinh^{-1}u,\xi')}{\sqrt{(\xi^2-\xi'^2-r^2)^2+(2\xi\xi')^2}}-\right]\xi'\,d\xi' \\
&+ \quad \text{etc.}
\end{aligned}
\tag{53}
$$

where

$$
u = \frac{\xi^2-\xi'^2-r^2}{2\xi\xi'} \ .
$$

It is evident that each multipole term has its own distinguishing angular (θ) and radial (r) dependence.

VI. RADIATION: PHYSICAL RELATION TO ITS SOURCE

The radiation expressed by Eqs.(50)-(54) establishes the link between the accelerated sources and what is measured and recorded in an expanding inertial reference frame. The measured observations yield very detailed knowledge about each accelerated source (in I and II) individually as well as about their relation to each other.

A. Fidelity

The most striking aspect of the radiation process is the fidelity of the signal measured in F. To bring this fidelity into sharper focus, consider the radiation from two localized multipole sources, one localized at $\xi'=\xi'_I$ and the other at $\xi'=\xi'_{II}$. Their mth multipole moments are therefore

$$
S^m_{I,II}(\tau',\xi') = S^m_{I,II}(\tau')\frac{\delta(\xi'-\xi'_{I,II})}{\xi'} \quad ,
$$

Consequently, the multipole superposition has the form

$$
\begin{aligned}
\psi_F(\xi,\tau,r,\theta) = & \left[\frac{2S_I^0(\tau+\sinh^{-1}u_I)}{\sqrt{(\xi^2-\xi_I'^2-r^2)^2+(2\xi\xi_I')^2}} - \frac{2S_{II}^0(\tau-\sinh^{-1}u_{II})}{\sqrt{(\xi^2-\xi_{II}'^2-r^2)^2+(2\xi\xi_{II}')^2}}\right] \\
& - e^{i\theta}\frac{\partial}{\partial r}\left[\frac{2S_I^1(\tau+\sinh^{-1}u_I)}{\sqrt{(\xi^2-\xi_I'^2-r^2)^2+(2\xi\xi_I')^2}} - \frac{2S_{II}^1(\tau-\sinh^{-1}u_{II})}{\sqrt{(\xi^2-\xi_{II}'^2-r^2)^2+(2\xi\xi_{II}')^2}}\right] \\
& + e^{2i\theta}\left(\frac{\partial^2}{\partial r^2}-\frac{1}{r}\frac{\partial}{\partial r}\right)\left[\frac{2S_I^2(\tau+\sinh^{-1}u_I)}{\sqrt{(\xi^2-\xi_I'^2-r^2)^2+(2\xi\xi_I')^2}} - \frac{2S_{II}^2(\tau-\sinh^{-1}u_{II})}{\sqrt{(\xi^2-\xi_{II}'^2-r^2)^2+(2\xi\xi_{II}')^2}}\right] \\
& + \quad \text{etc.}
\end{aligned} \tag{54}
$$

where

$$
u_{I,II} = \frac{\xi^2-\xi_{I,II}'^2-r^2}{2\xi\xi_{I,II}'} \ .
$$

Compare Eq.(48) with Eq.(54). The temporal evolution of every localized accelerated multipole source

$$
S_{I,II}^m(\tau',\xi') \quad m=0,\pm1,\pm2,\cdots
$$

displays itself with 100% fidelity as the correspondingly measurable amplitude

$$
S_{I,II}^m(\tau\pm\sinh^{-1}u_{I,II}) \quad m=0,\pm1,\pm2,\cdots
$$

on the hypersurface of synchronous time $\xi = const$ of the expanding inertial observation frame in F. There is no distortion and no spatial chirp (τ-dependent redshift), regardless how violently the localized multipole source got accelerated.

As pointed out in section III, the high fidelity is due to the *expanding* nature of the *inertial observation frame.* Such a frame consists of an expanding set of free float recording clocks with radio receivers all synchronized and coherently phased to measure the complex amplitude (magnitude and phase) of the spatial amplitude profile at any fixed synchronous time $\xi > 0$. Once these recording clocks have been brought into existence, they can always be used to measure, receive, and record the e.m. field with 100% fidelity.

Not so for the usual *static inertial observation frame*, which consist of a static lattice of free float meter rods, clocks, and radio receivers. Such a frame would be entirely unsuitable for observing the emission of radiation from violently accelerated bodies. Once the recording clocks have been assembled by the physicist/observer into such a frame, the reception, measurement, and recording of electromagnetically encoded information will always be compromised by the destructive blueshift from the accelerated source.

B. Spatial Structure of the Source

The second striking feature of the emitted radiation is that its measurement yields the spatial multipole structure of the source. Measure the angular distribution for a given radial coordinate r in the plane transverse to the direction of acceleration. Do a least squares fit to the measured data points in order to determine each of the Fourier coefficients in Eq.(52). In order to obtain the radial distribution of each of these coefficients, repeat this determination for various r values. A second least squares analysis yields the radial derivatives and hence the amplitude of each multipole moment $S_{I,II}^m$, $m=0,\pm1,\pm2,\cdots$ in Eq.(54).

C. Double Slit Interference

The third property of the radiation process is that it highlights the interference between the waves coming from Rindler sectors I and II. The interference pattern, which is recorded on a hypersurface of synchronous time $\xi =$ *constant*, has fringes whose separation yields the separation between the two localized in I and II. Let these sources be located symmetrically at

$$
\xi_I' = \xi_{II}' \equiv \xi_0' \ ,
$$

and let them have equal proper frequency ω_0 and hence (in compliance with the first term of the wave Eq.(27) equal Rindler coordinate frequency

$$\omega = \omega_0 \xi_0' \ .$$

Consequently, they are characterized by their amplitudes and their phases. Indeed, their form is

$$\begin{aligned} S_I^m(\tau + \sinh^{-1} u_I) &= A_I^0 \cos[\omega_0 \xi_0'(\tau + \sinh^{-1} u_I) + \delta_I^m] \\ S_{II}^m(\tau - \sinh^{-1} u_{II}) &= A_{II}^0 \cos[\omega_0 \xi_0'(\tau - \sinh^{-1} u_I) + \delta_{II}^m] \ . \end{aligned} \tag{55}$$

Thus the full scalar field, Eq.(54), expresses two waves. Both propagate in the expanding inertial frame, which is coordinatized by (ξ, τ, r, θ). Their respective wave crests are located in compliance with the constant phase conditions $\tau \pm \sinh^{-1} u_I = const.$ Consequently, one wave travels into the $+\tau$-direction with amplitude A_I^m, the other into the $-\tau$-direction with amplitude A_{II}^m. They have well-determined phase velocities. Together, these two waves form an interference pattern of standing waves,

$$\begin{aligned} \psi_F(\xi, \tau, r, \theta) = \frac{1}{\sqrt{(\xi^2 - \xi_0'^2 - r^2)^2 + (2\xi\xi_0')^2}} \Bigg[& (A_I^0 - A_{II}^0) \cos[\omega_0 \xi_0'(\tau - \sinh^{-1} u_I) + \delta_I^0] \\ & - 2A_{II}^0 \sin\left(\omega_0 \xi_0' \tau + \frac{\delta_{II}^0 + \delta_I^0}{2}\right) \sin\left(\omega_0 \xi_0' \sinh^{-1} u_I - \frac{\delta_{II}^0 - \delta_I^0}{2}\right) \Bigg] \\ + \quad & \text{higher multipole terms of order } m = 1, 2, 3, \cdots \ . \end{aligned} \tag{56}$$

The amplitude of this interference pattern is $A_{II}^0 > 0$, and there is a uniform background of amplitude $(A_I^0 - A_{II}^0) > 0$. At synchronous time ξ the interference fringes along the τ-direction can be read off the factor

$$\sin(\omega_0 \xi_0' \tau + \frac{\delta_{II}^0 + \delta_I^0}{2})$$

in Eq.(56). Consequently, the fringes are spaced by the amount

$$\begin{pmatrix} \text{proper} \\ \text{fringe spacing} \end{pmatrix} = \frac{2\pi\xi}{\omega_0 \xi_0'} = \frac{\xi}{\xi_0'} \times \frac{1}{\begin{pmatrix} \text{proper frequency} \\ \text{of the source} \end{pmatrix}} = \frac{2\xi}{\text{source separation}} \times \frac{1}{\begin{pmatrix} \text{proper frequency} \\ \text{of the source} \end{pmatrix}} \ .$$

This means that, analogous to a standard optical interference pattern, the fringe spacing is inversely proportional to the distance $2\xi_0'$ between the two sources. Furthermore, the position of this interference pattern depends on the phase of source I relative to source II [19]. It is difficult to find a more welcome way than the four Rindler sectors for double slit interference.

These observations lead to the conclusion that (i) the four Rindler sectors quite naturally accommodate a *double slit interferometer*, and that (ii) the spatial as well as the temporal properties of the interference fringes, together with the magnitude of the travelling background wave, are enough to reconstruct every aspect of the two sources, Eq.(55).

D. The Rindler Interferometer

The double slit interferometer works as follows: A plane wave which starts in Rindler sector P gets split into two partial components which propagate through I and II. There they get modified by the two respective scatterers. They are two pointlike dipole loops accelerating into opposite directions. Each loop acts as a transmitter which re-radiates the electromagnetic field from the impinging wave. The e.m. fields emitted by these two transmitters exits through the event horizons of I and II. Upon recombining in F they produce an interference pattern as measured on the hypersurface $\xi = constant$ of the expanding inertial observation frame. The strength and the variations in this pattern are determined by (i) the proper separation between the two scatterers, (ii) their relative strengths and (iii) their relative phase. In fact, from this interference pattern one can reconstruct the currents $\dot{q}_I(\tau')$ and $\dot{q}_{II}(\tau')$, including the amplitudes, phases for each of them. In brief, the *expanding* inertial observation frame is the "screen" on which one can literally "see" what is going on in each of the two accelerated frames I and II.

VII. RADIATED POWER

The electric and magnetic field components are obtained from the wave function $\psi_F(\xi, \tau, r, \theta)$ by taking the partial derivatives listed in the tables in Section II.C. With their help we shall now find the Poynting vector component along the τ-direction, namely

$$\frac{1}{4\pi}(\hat{B}_r\hat{E}_\theta - \hat{B}_\theta\hat{E}_r)\xi \equiv T^{\xi}_{\ \tau} \ .$$

Its space integral,

$$\int_0^\infty \int_0^\infty \int_0^{2\pi} T^{\xi}_{\ \tau}\, \xi d\tau\, r dr\, d\theta \ , \tag{57}$$

is the total radiated momentum (=radiant energy flow) into the τ-direction. It is positive (resp. negative) whenever the source is confined to Rindler sector I (resp. II). Furthermore, the τ-momentum is independent of the synchronous time ξ because τ is a cyclic coordinate. This τ-momentum measures the energy radiated by the two accelerated sources, and it takes the place of what in a static inertial reference frame is the emitted energy.

Both the T.E. and the T.M. field have the same Poynting vector component along the τ-direction. More precisely, reference to the table of T.E. and the T.M. field components (Section II) shows that in Rindler sector F this Poynting object is

$$T^{\xi}_{\ \tau} = \frac{\xi}{4\pi}\left[\frac{\partial}{\partial\xi}\left(\frac{\partial\psi}{\partial r}\right)\frac{1}{\xi}\frac{\partial}{\partial\tau}\left(\frac{\partial\psi}{\partial r}\right) + \frac{\partial}{\partial\xi}\left(\frac{1}{r}\frac{\partial\psi}{\partial\theta}\right)\frac{1}{\xi}\frac{\partial}{\partial\tau}\left(\frac{1}{r}\frac{\partial\psi}{\partial\theta}\right)\right] \ , \tag{58}$$

the same for both types of fields. Furthermore, the wave function ψ is governed by a wave equation, which is also common to both fields. Consequently, the mathematical analysis which relates observations to the radiation sources is the same for both types of radiation fields. However, it is the difference in two types of sources which is important from the viewpoint of physics.

The only difference lies in the source and hence in the amplitude and phase of ψ in F. Comparing the ensuing Eq.(60) with Eq.(63), one sees that T.E. and T.M. polarized radiation are caused by the densities of magnetic and electric dipole moment respectively.

A. Axially Symmetric Source and Field

The simplest nontrivial sources for the inhomogeneous wave Eq.(35) is those which are axially symmetric. For T.E. radiation magnetic dipoles are the most important sources, while for T.M. radiation they are electric dipoles.

1. Magnetic Dipole and its Radiation Field

Consider radiation emitted from two magnetic dipoles. Have them be two circular loop antennas each of area of πa^2 aligned parallel to the (x, y)-plane with center on the z-axis. Fix their location in Rindler sectors I and II by having them located at $\xi' = \xi'_I$ and $\xi' = \xi'_{II}$ so that they are accelerated into opposite directions. Suppose each antenna has proper current

$$i_{I,II}(\tau') = \frac{1}{\xi'_{I,II}}\frac{d\, q_{I,II}(\tau')}{d\tau'} \qquad \left[\frac{\text{charge}}{\text{length}}\right] \ .$$

Then its magnetic moment is

$$i_{I,II}(\tau')\pi a^2 \equiv \mathbf{m}_{I,II}(\tau') \ , \qquad \Big[\ (\text{proper current}) \times (\text{length})^2 = \text{dipole moment}\ \Big]$$

its charge-flux four-vector obtained from Eq.(29) is

$$\begin{aligned}\left(\hat{S}_{\tau'}, \hat{S}_{\xi'}, \hat{S}_{r'}, \hat{S}_{\theta'}\right) &= \left(0, 0, \frac{1}{r'}\frac{\partial S}{\partial\theta'}, \frac{\partial S}{\partial r'}\right) \\ &= i_{I,II}(\tau')\delta(\xi' - \xi'_{I,II})\delta(r' - a)\,(0,0,0,-1) \ , \qquad \left[\frac{\text{charge}}{\text{length}^3}\right]\end{aligned} \tag{59}$$

and the corresponding scalar source function S for Eq.(27) is

$$S: \quad S_{I,II}(\tau', \xi', r', \theta') = \frac{d\, q_{I,II}(\tau')}{d\tau'}\frac{\delta(\xi' - {\xi'}_{I,II})}{\xi'}\,\Theta(r' - a) \qquad \left[\frac{\text{charge}}{\text{length}^2}\right] \ . \tag{60}$$

Here Θ is the Heaviside unit step function. The proper magnetic dipole is the proper volume integral of this source,

$$\int_0^\infty \int_0^\infty \int_0^{2\pi} S_{I,II}(\tau',\xi',r',\theta')d\xi' r' dr' d\theta' = \pi a^2 i_{I,II}(\tau') \qquad \text{[(area) } \times \text{ (proper current)]}$$

Being symmetric around its axis, such a source produces only radiation which is independent of the polar angle θ. Consequently, all partial derivatives w.r.t. θ vanish, and the full scalar field, Eq.(54), in F becomes with the help of Eq.(47)

$$\psi_F(\xi,\tau,r,\theta) = 2\pi a^2 \left[\frac{1}{\sqrt{(\xi^2 - \xi_I'^2 - r^2)^2 + (2\xi\xi_I')^2}} \frac{dq_I(\tau + \sinh^{-1} u_I)}{d\tau} - \frac{1}{\sqrt{(\xi^2 - \xi_{II}'^2 - r^2)^2 + (2\xi\xi_{II}')^2}} \frac{dq_{II}(\tau - \sinh^{-1} u_{II})}{d\tau} \right] , \qquad \text{[charge]} \tag{61}$$

where

$$u_{I,II} = \frac{\xi^2 - \xi_{I,II}'^2 - r^2}{2\xi\xi_{I,II}'}$$

This is the T.E. scalar field due to a localized pair of axially symmetric loop antennas, each one with its own time dependent current. By setting one of them to zero one obtains the radiation field due to the other.

2. Electric Dipole and its Radiation Field

The most important electric dipole radiators are two linear antennas each of length a aligned parallel to the z-axis, located at $\xi' = \xi_I'$ and $\xi' = \xi_{II}'$ located in Rindler sectors I and II, and hence accelerated into opposite directions. Suppose each antenna has electric dipole moment

$$q_{I,II}(\tau')\, a \equiv \mathbf{d}_{I,II}(\tau') \qquad \text{[(charge) } \times \text{ (length) = dipole moment]}$$

Its charge-flux four-vector obtained from Eq.(31) is

$$\begin{aligned}\left(\hat S_{\tau'}, \hat S_{\xi'}, \hat S_{r'}, \hat S_{\theta'}\right) &= \left(\frac{\partial S}{\partial \xi'}, \frac{1}{\xi'}\frac{\partial S}{\partial \tau'}, 0, 0, \right) \\ &= \left(q_{I,II}(\tau') a \frac{d}{d\xi'}\delta(\xi' - \xi_{I,II}'), \frac{1}{\xi'}\frac{dq(\tau')}{d\tau'} a\, \delta(\xi' - \xi_{I,II}'), 0, 0 \right) \frac{\delta(r'-0)\delta(\theta'-\theta_0')}{r'} \; , \qquad \left[\frac{\text{charge}}{\text{length}^3}\right]\end{aligned} \tag{62}$$

and the corresponding scalar source function S for Eq.(27) is therefore

$$S: \quad S_{I,II}(\tau',\xi',r',\theta') = q_{I,II}(\tau') a\; \delta(\xi' - \xi_{I,II}) \frac{\delta(r'-0)\delta(\theta'-\theta_0')}{r'} \qquad \left[\frac{\text{charge}}{\text{length}^2}\right] . \tag{63}$$

This source is symmetric around the z-axis because it is non-zero only at $r' = 0$. The electric dipole moment is the proper volume integral of this source,

$$\int_0^\infty \int_0^\infty \int_0^{2\pi} S_{I,II}(\tau',\xi',r',\theta')d\xi' r' dr' d\theta' = q_{I,II}(\tau') a \qquad \text{[(charge) } \times \text{ (length)]}$$

The axial symmetry of the source implies that its radiation is independent of the polar angle θ. Consequently, except some for a source-dependent factor, the scalar field ψ_F in F is the same as Eq.(61). One finds

$$\psi_F(\xi,\tau,r,\theta) = 2a \left[\frac{\xi_I'}{\sqrt{(\xi^2 - \xi_I'^2 - r^2)^2 + (2\xi\xi_I')^2}}\; q_I(\tau + \sinh^{-1} u_I) - \frac{\xi_I'}{\sqrt{(\xi^2 - \xi_{II}'^2 - r^2)^2 + (2\xi\xi_{II}')^2}}\; q_{II}(\tau - \sinh^{-1} u_{II}) \right] . \qquad \text{[charge]} \tag{64}$$

This is the T.M. scalar field due to a pair of localized linear antennas, both situated on the z-axis, each one with its own time-dependent dipole moment $q(\tau')\,a$. By setting one of them to zero one obtains the radiation field due to the other.

B. Flow of Radiant T.E. Field Energy

Any loop antenna radiates only for a finite amount of time. Consequently, one can calculate the flow of total emitted energy, which is given by the spatial integral, Eq.(57). The fact that θ is a cyclic coordinate for axially symmetric sources and radiation implies that $\hat{E}_r = \hat{B}_\theta = 0$ for the T.E. field. Consequently, the spatial integral, a conserved quantity independent of time ξ, reduces with the help of the table of derivatives in Section IV to

$$\int_{-\infty}^{\infty}\int_0^{\infty}\int_0^{2\pi} T^{\xi}_{\ \tau}\,\xi d\tau\, rdr\, d\theta = \int_{-\infty}^{\infty}\int_0^{\infty}\int_0^{2\pi} \frac{1}{4\pi}\hat{B}_r \times \hat{E}_\theta \xi\, \xi d\tau\, rdr\, d\theta \tag{65}$$

$$= \frac{1}{4\pi}\int_0^{\infty}\int_0^{\infty}\int_0^{2\pi} \frac{1}{\xi}\frac{\partial}{\partial r}\frac{\partial \psi_F}{\partial \tau} \times \frac{\partial}{\partial r}\frac{\partial \psi_F}{\partial \xi}\xi\, \xi d\tau\, rdr\, d\theta$$

$$= (\pm)\frac{(\pi a^2)^2}{\xi'^4_{I,II}}\int_{-\infty}^{\infty}\frac{2}{3}\left\{\left(\frac{d^3 q_{I,II}(\tau)}{d\tau^3}\right)^2 + \left(\frac{d^2 q_{I,II}(\tau)}{d\tau^2}\right)^2\right\} d\tau \tag{66}$$

The computation leading to the last line has been consigned to the Appendix. This computed quantity is the total energy flow (energy $\times$ velocity), or equivalently, the total momentum into the τ-direction, radiated by a magnetic dipole accelerated uniformly in Rindler sector I (upper sign) or in Rindler sector II (lower sign).

The full scalar radiation field

$$\psi_F(\xi,\tau,r,\theta) = 2\pi a^2 \frac{\pm 1}{\sqrt{(\xi^2 - \xi'^2_I - r^2)^2 + (2\xi\xi'_I)^2}}\frac{dq_{I,II}(\tau')}{d\tau}\ , \tag{67}$$

with

$$\tau' = \tau \pm \sinh^{-1}\frac{\xi^2 - \xi'^2_{I,II} - r^2}{2\xi\xi'_{I,II}}\ ,$$

is a linear functional correspondence. It maps the temporal history $q_{I,II}(\tau')$ at $\xi' = \xi'_{I,II}$ in Rindler sector I (resp. II) with 100% fidelity onto a readily measurable e.m. field along the τ-axis (or a line parallel to it) on the spatial hypersurface $\xi = const.$ in Rindler sector F. This correspondence has 100% fidelity because, aside from a τ-independent factor, the source history $q_{I,II}(\tau')$ and the scalar field $\psi_F(\tau)$ differ only by a constant τ-independent shift on their respective domains $-\infty < \tau' < \infty$ and $-\infty < \tau < \infty$. This implies that

$$\frac{\partial}{\partial \tau'} = \frac{\partial}{\partial \tau} \tag{68}$$

when applied to the source function $q_{I,II}$. The expression for the radiated momentum becomes more transparent physically if one uses the proper time derivative

$$\frac{d}{dt'} \equiv \frac{1}{\xi'}\frac{\partial}{\partial \tau'} = \frac{1}{\xi'}\frac{\partial}{\partial \tau}$$

at the source. Introduce the (proper) magnetic moment of the current loop having radius a:

$$\mathbf{m} = \pi a^2 \frac{1}{\xi'}\frac{\partial q}{\partial \tau'}\ .$$

One finds from Eq.(66) that the proper radiated longitudinal momentum (i.e. physical, a.k.a. orthonormal, component of energy flow pointing into the τ-direction) measured per proper spatial τ-interval $\xi\, d\tau$ in F is

$$\mathcal{I}_{T.E.} = (\pm)\ \frac{\xi'^2}{\xi^2}\frac{2}{3}\left[\left(\frac{d^2\mathbf{m}}{dt'^2}\right)^2 + \frac{1}{\xi'^2}\left(\frac{d\mathbf{m}}{dt'}\right)^2\right]\ . \tag{69}$$

This is the formula for the proper radiant energy flow due to a magnetic dipole moment subject to uniform linear acceleration $1/\xi'$. There are two factors of $1/\xi$. The the first converts the coordinate τ-momentum component into its physical component. The second is due to the fact that Eq.(69) expresses this quantity per proper distance into the τ direction.

When the acceleration $1/\xi'$ is small then the second term becomes small compared to the first. In fact, one recovers the familiar Larmor formula [5] relative to a *static inertial* frame by letting $\xi = \xi'$ and letting $1/\xi' \to 0$, which corresponds to inertial motion. By contrast, Eq.(69) is the correct formula for an *accelerated* dipole moment. However, observation of radiation from such a source entails that the measurements be made relative to an *expanding inertial* reference frame.

C. Flow of Radiant T.M. Field Energy

The mathematical computation leading from Eq.(65) and ending with Eq.(69) can be extended without any effort to T.M. radiation. The extension consists of replacing a T.E. source with a corresponding T.M. source,

$$\frac{1}{\xi'}\frac{\partial q}{\partial \tau'}\pi a^2 \to qa \ ,$$

or equivalently

$$\mathbf{m} \to \mathbf{d} \ .$$

Consequently, the formula for the flow of T.M. radiant energy due to an electric dipole subject to uniform linear acceleration $1/\xi'$ is

$$\mathcal{I}_{T.M.} = (\pm)\ \frac{\xi'^2}{\xi^2}\frac{2}{3}\left[\left(\frac{d^2\mathbf{d}}{dt'^2}\right)^2 + \frac{1}{\xi'^2}\left(\frac{d\mathbf{d}}{dt'}\right)^2\right] \ . \tag{70}$$

The justification for this extension is Eqs.(58) and (34). They are the same for T.E. and T.M. radiation. The radiation intensity expressed by Eq.(70) extends the familiar Larmor formula for radiation from an *inertially moving* electric dipole [5] to one which is *accelerated* linearly and uniformly.

VIII. VIOLENT ACCELERATION

The second term in the radiation formula, Eq.(69), is new. Under what circumstance does it dominate? Consider the circumstance where the magnetic dipole oscillates with proper frequency $\omega_0 = 2\pi/\lambda_0$. By averaging the emitted radiation over one cycle, one finds

$$(\pm)\ \frac{\xi'^2}{\xi^2}\frac{2}{3}\left[\left(\frac{d^2\mathbf{m}}{dt'^2}\right)^2 + \frac{1}{\xi'^2}\left(\frac{d\mathbf{m}}{dt'}\right)^2\right] \longrightarrow (\pm)\ \frac{\xi'^2}{\xi^2}\frac{\omega_0^4}{3}\langle\mathbf{m}^2\rangle\left[1+\frac{1}{\xi'^2\omega_0^2}\right] = (\pm)\ \frac{\xi'^2}{\xi^2}\frac{\omega_0^4}{3}\langle\mathbf{m}^2\rangle\left[1+\frac{1}{(2\pi)^2}\frac{\lambda_0^2}{\xi'^2}\right]$$

Thus the criterion for "violent" acceleration is that its inverse, the Fermi-Walker length of the accelerated point object be small compared to the emitted wavelength,

$$\frac{\lambda_0}{\xi'} \equiv \frac{\lambda_0 \times (\text{proper acc'n})}{c^2} \gg 2\pi$$

or equivalently

$$\frac{(\text{proper acc'n})}{c} \gg \omega_0 \ . \tag{71}$$

Recall that $c/$(proper acceleration) is the time it takes for the oscillator to acquire a relativistic velocity relative to an inertial frame. Also recall that $2\pi/\omega_0$ is the time for one oscillation cycle. Consequently, the criterion for "violence" is that

$$\begin{pmatrix}\text{time for oscillator}\\ \text{to acquire}\\ \text{relativistic velocity}\end{pmatrix} \ll (\text{oscillation period}) \ . \tag{72}$$

When this condition is fulfilled, the Larmor contribution to the radiation is eclipsed by the Rindler contribution.

IX. RADIATIVE VS. NONRADIATIVE MOMENERGY

Consider a dipole moment, **m** or **d**, which is time-independent in its own accelerated frame. The augmented Larmor formula, Eq.(70) and (69), yields zero radiative τ-momentum relative the expanding inertial frame in Rindler sector F:

$$\int_0^\infty \int_0^\infty \int_0^{2\pi} T^{\xi}{}_{\tau}\, \xi d\tau\, r dr\, d\theta = 0\ . \tag{73}$$

However, for a non-zero static dipole moment the other momenergy[1] components, also measured in F, are non-zero:

$$\int_0^\infty \int_0^\infty \int_0^{2\pi} T^{\xi}{}_{\xi}\, \xi d\tau\, r dr\, d\theta \neq 0 \tag{74}$$

$$\int_0^\infty \int_0^\infty \int_0^{2\pi} T^{\xi}{}_{r}\, \xi d\tau\, r dr\, d\theta \neq 0 \tag{75}$$

$$\int_0^\infty \int_0^\infty \int_0^{2\pi} T^{\xi}{}_{\theta}\, \xi d\tau\, r dr\, d\theta = 0 \qquad \text{(“axial symmetry”)} \tag{76}$$

Equations (73)-(76) express an observationally and hence conceptually precise distinction between the radiative and non-radiative e.m. fields of a dipole source accelerated in Rindler sector I: The augmented Larmor formula implies that the dipole emits radiation if and only if its τ-momentum, the spatial integral of $T^{\xi}{}_{\tau}$ in the expanding inertial frame, is non-zero. Furthermore, the existence of a dipole field, static in Rindler sector I, is expressed by the non-vanishing of the other momenergy components, Eqs.(74)-(75). Like the τ-momentum, these components are also measurable in the expanding inertial frame. If the dipole is not static then the the emitted radiation gets tracked by the τ-momentum. In that case the other momenergy components play an auxiliary role. They only track the sum of static dipole field and the radiative field, not the separate contributions.

X. UNIFYING PERSPECTIVE

The four Rindler sectors lend themselves to a unifying perspective. The context which makes this possible is the emission and observation of radiation from a body accelerated linearly and uniformly. The requirement that signals be transmitted with 100% fidelity implies that the spacetime arena for this radiation process consist of two adjacent Rindler coordinatized sectors, such as I and F or II and F.

The unifying perspective applied to two adjacent Rindler sectors is brought into a particularly sharp focus by the augmented Larmor formula, Eqs.(69) or (70). This is because the physical basis of this formula is a radiation process which starts in one of the two Rindler sectors and ends in the other. Indeed, the radiative longitudinal momentum (longitudinal flow of radiative energy) observed and measured in the expanding inertial frame in F is expressed directly in terms of the behaviour of the dipole source in the accelerated frame in I (or II).

The unifying perspective applies to all four Rindler sectors if one considers a scattering process which starts in P and ends in F. In such a process an e.m. wave starts in Rindler sector P, splits into two partial waves which cross the past event horizons and enter the respective Rindler sectors I and II. The partial wave in I excites the internal degree of freedom of the dipole oscillator accelerated in I. There the oscillations constitute a source for the scattered radiation which propagates into F. The other partial wave, which propagates through Rindler sector II, also reaches F. There the resultant interference pattern is observed and measured. It is evident that this interference pattern is made possible by the properties of the four Rindler sectors combined, the Rindler interferometer of Section VI D.

Thus both the Rindler interferometer and the augmented Larmor formula provide a unifying perspective. It joins adjacent Rindler coordinate charts into a single spacetime arena for radiation and scattering processes from accelerated bodies.

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[1]The word *momenergy*, a term first coined by J.A. Wheeler, refers to the single concept which ordinarily is referred to by the cumbersome word “energy-momentum”. Given the fact that, in the physics of particles and fields, the concepts “momentum” and “energy” are merely different aspects united by a change of inertial frames into a new single concept, the case for correspondingly uniting the compound word “ momentum-energy”, or “energy-momentum” into the single word “momenergy” is appropriate.

This perspective is at variance with a philosophy which seeks a particle-antiparticle definition in non-rectilinear coordinate systems in flat spacetime [20].

Such a philosophy typically focuses on one of the Rindler charts to the exclusion of all the others. The application of quantum field theory to such a chart leads to the paradox of spurious particle production in flat spacetime. As a result quantum theory remains meaningfully invariant only under a subset of classically allowed coordinate transformations.

A proposed solution is to disallow – in quantum theory – a large class of coordinate transformations, such as those leading to Rindler charts F or I [20].

However, the fault does not lie with these Rindler charts. Instead, it lies with the underlying philosophy which seeks a definition of the particle-antiparticle concept in one of the coordinate charts while ignoring reference to the others. Such a selective focus does not comply with, and hence is forbidden by the unifying perspective implied by the augmented Larmor formula and by the Rindler double-slit interferometer.

XI. CONCLUSION

The subject of this article is the physics of accelerated frames. The theme is: "How does one observe and measure the properties of violently accelerated bodies?" Answering this question has led to three results.

First of all, suppose one considers the transmission of e.m. signals from a translationally accelerated transmitter to a receiver in an inertial frame. One finds that the signals can be transmitted without any time dependent Doppler distortion and with 100% fidelity *provided* the signals are received in an inertial frame which is *expanding.* A static inertial frame would not do.

Second, Larmor's radiation formula for the radiation intensity from a dipole source gets augmented if that dipole is subjected to linear uniform acceleration. Under this circumstance the total radiation is the sum of the magnetic and electric dipole radiation,

$$\mathcal{I}_{total} = (\pm)\,\frac{\xi'^2}{\xi^2}\left\{\frac{2}{3}\left[\left(\frac{d^2\mathbf{d}}{dt'^2}\right)^2+\frac{1}{\xi'^2}\left(\frac{d\mathbf{d}}{dt'}\right)^2\right]+\frac{2}{3}\left[\left(\frac{d^2\mathbf{m}}{dt'^2}\right)^2+\frac{1}{\xi'^2}\left(\frac{d\mathbf{m}}{dt'}\right)^2\right]\right\}\,. \tag{77}$$

The amount of that augmentation becomes dominant when the acceleration is so large that its "Fermi-Walker" length ($\xi' = c^2/\text{acceleration}$) is smaller than the wavelength of the emitted radiation, or equivalently, when Eq.(72) is fulfilled.

Finally, taking note of the fact that for every source accelerated to the right there is a twin source accelerated to the left, suppose these two sources are irradiated coherently. Then these twins together with their concomitant expanding inertial reference frame form an interferometer. More precisely, the four Rindler coordinatized sectors form an interferometer, the Rindler interferometer. Its geometrical arena consists of the Rindler sectors $P \rightarrow (I, II) \rightarrow F$ as exhibited in Figure 1. They accommodate what in Euclidean space would be the components of an optical interferometer with (i) P serving as the beam splitter, (ii) the spacetimes of the two accelerated frames I and II serving as the two arms, and (iii) the spacetime F serving as the beam re-combiner. The interference pattern is recorded by the expanding inertial frame in F.

XII. ACKNOWLEDGEMENT

The author would like to thank Nirmala Prakash and Yuri Obukhov for helpful remarks.

XIII. APPENDIX: POTENTIAL, FIELD AND RADIATED MOMENTUM OF AN ACCELERATED POINT-LIKE MAGNETIC DIPOLE MOMENT

Even though the focus of this article is on the T.E. radiation from an axially symmetric circular dipole antenna, the mathematical structure of T.M. radiation is virtually the same for both. One merely has to interchange the magnetic with the electric field components to obtain one from the other. Furthermore, both are derived from a single scalar which obeys the same inhomogeneous scalar wave Eq.(5). For T.E. radiation from a magnetic dipole localized at $r' = 0$ and $\xi' = constant$ in Rindler sector I (upper sign) or II (lower sign), this scalar is the simplified version of Eq.(61):

$$\psi_F(\xi,\tau,r,\theta) = \frac{2\pi a^2}{2\xi\xi'}\frac{\pm 1}{\sqrt{u^2+1}}\frac{dq(\tau \pm \sinh^{-1} u)}{d\tau} \tag{78}$$

where

$$u=\frac{\xi^2-\xi'^2-r^2}{2\xi\xi'}$$

What are the T.E. field components and what is the Poynting integral, Eq.(65)? With some care the calculation is reasonably straight forward. We exhibit the calculational path in this appendix because it probably is optimal and hence also useful for the case of T.M. radiation.

A. Vector Potential

For T.E. radiation the components of the vector potential relative to the orthonormal basis are

$$\begin{aligned}(\hat{A}_\xi,\hat{A}_\tau,\hat{A}_r,\hat{A}_\theta)&=\left(0,0,\frac{-1}{r}\frac{\partial\psi_F}{\partial\theta},\frac{\partial\psi_F}{\partial r}\right)\\&=\left(0,0,\quad 0\quad,\frac{\pm4\pi a^2 r}{(2\xi\xi')^2}\left(\frac{u}{(u^2+1)^{3/2}}\dot{q}\mp\frac{1}{u^2+1}\ddot{q}\right)\right)\end{aligned}\tag{79}$$

where an over-dot refers to the partial derivative

$$\dot{q}=\frac{\partial q(\tau\pm\sinh^{-1}u)}{\partial\tau}\ .$$

B. Field Components

In compliance with the table of derivatives in Section IV D 1, the relevant non-zero component of the electric field is

$$\begin{aligned}\hat{E}_\theta\equiv\hat{F}_{\theta\xi}&=\frac{1}{r}\left(\frac{\partial A_\xi}{\partial\theta}-\frac{\partial A_\theta}{\partial\xi}\right)\\&=-\frac{\partial}{\partial\xi}\frac{\partial\psi_F}{\partial r}\ ,\end{aligned}\tag{80}$$

which with the help of Eq.(79) becomes

$$\begin{aligned}\hat{E}_\theta&=-(\mp)4\pi a^2\ \ r\frac{\partial}{\partial\xi}\left[\frac{1}{(2\xi\xi')^2}\left(\frac{u}{(u^2+1)^{3/2}}\dot{q}\mp\frac{1}{u^2+1}\ \ddot{q}\right)\right]\\&=2\pi a^2(\alpha\dot{q}+\beta\ddot{q}+\gamma\ \dddot{q})\ ,\end{aligned}\tag{81}$$

where

$$\begin{aligned}\alpha&=\frac{\mp2r}{(2\xi\xi')^2}\left(\frac{-3}{\xi}\frac{u}{(u^2+1)^{5/2}}+\frac{1}{\xi'}\frac{1-2u^2}{(u^2+1)^{5/2}}\right)\\\beta&=\frac{-2r}{(2\xi\xi')^2}\left(\frac{1}{\xi}\frac{2-u^2}{(u^2+1)^2}+\frac{3}{\xi'}\frac{u}{(u^2+1)^2}\right)\\\gamma&=\frac{\mp2r}{(2\xi\xi')^2}\left(\frac{1}{\xi}\frac{u}{(u^2+1)^{3/2}}-\frac{1}{\xi'}\frac{1}{(u^2+1)^{3/2}}\right)\ .\end{aligned}\tag{82}$$

Similarly the relevant non-zero magnetic field component is

$$\begin{aligned}\hat{B}_r\equiv\hat{F}_{\theta\tau}&=\frac{1}{r\xi}\left(\frac{\partial A_\tau}{\partial\theta}-\frac{\partial A_\theta}{\partial\tau}\right)\\&=-\frac{1}{r\xi}\frac{\partial}{\partial\tau}\left(r\frac{\partial\psi_F}{\partial r}\right)=-\frac{1}{\xi}\frac{\partial}{\partial\tau}\frac{\partial\psi_F}{\partial r}\ ,\end{aligned}\tag{83}$$

which with the help of Eq.(79) becomes

$$\hat{B}_r = 2\pi a^2(\delta\ddot{q} + \epsilon \dddot{q}) \ , \tag{84}$$

where

$$\begin{aligned} \delta &= \frac{\mp 2r}{(2\xi\xi')^2}\frac{1}{\xi}\frac{u}{(u^2+1)^{3/2}} \\ \epsilon &= \frac{2r}{(2\xi\xi')^2}\frac{1}{\xi}\frac{1}{u^2+1} \ . \end{aligned} \tag{85}$$

C. Radiated Momentum

Being generated by a circular loop, the density of radiated momentum pointing into the $\pm\tau$ direction is independent of the polar angle θ. Consequently, that density's spatial integral, Eq.(66), reduces to

$$\begin{aligned} \int_{-\infty}^{\infty}\int_0^{\infty}\int_0^{2\pi} T^{\xi}_{\ \tau}\,\xi d\tau\, r dr\, d\theta &= \frac{1}{4\pi}\int_{-\infty}^{\infty}\int_0^{\infty}\int_0^{2\pi} \hat{B}_r \times \hat{E}_\theta \xi\, \xi d\tau\, r dr\, d\theta \\ &= \frac{(2\pi a^2)^2 2\pi}{4\pi}\int_{-\infty}^{\infty}\int_0^{\infty}(\delta\ddot{q} + \epsilon \dddot{q})(\alpha\dot{q} + \beta\ddot{q} + \gamma \dddot{q})\xi^2 d\tau\, r\, dr \end{aligned} \tag{86}$$

The τ-integration affects only the dotted factors, and they vanish outside a sufficiently large τ-interval, i.e. $\dot{q}(\pm\infty) = \ddot{q}(\pm\infty) = 0$. Consequently, integration by parts yields

$$\int_{-\infty}^{\infty} \ddot{q}\dot{q}\, d\tau = \int_{-\infty}^{\infty} \dddot{q}\,\ddot{q}\ \ d\tau = 0$$

and

$$\int_{-\infty}^{\infty} \dddot{q}\,\dot{q}\ \ d\tau = -\int_{-\infty}^{\infty} \ddot{q}^2 d\tau \neq 0$$

Thus there remain only three non-zero terms in the integral,

$$\int_{-\infty}^{\infty}\int_0^{\infty}\int_0^{2\pi} T^{\xi}_{\ \tau}\,\xi d\tau\, r dr\, d\theta = \frac{(2\pi a^2)^2 2\pi}{4\pi}\int_{-\infty}^{\infty}\int_0^{\infty}\left(\epsilon\gamma \dddot{q}^{\,2} + (\beta\delta - \alpha\epsilon)\ddot{q}^2\right)\xi^2 d\tau\, r\, dr \ . \tag{87}$$

The coefficients of the squared terms are

$$\epsilon\gamma = \mp\frac{4r^2}{(2\xi\xi')^4}\left(\frac{1}{\xi^2}\frac{u}{(u^2+1)^{5/2}} - \frac{1}{\xi\xi'}\frac{1}{(u^2+1)^{5/2}}\right) \tag{88}$$

$$\beta\delta = \mp\frac{4r^2}{(2\xi\xi')^4}\left(-\frac{1}{\xi^2}\frac{(2-u^2)u}{(u^2+1)^{7/2}} - \frac{3}{\xi\xi'}\frac{u^2}{(u^2+1)^{7/2}}\right) \tag{89}$$

$$\alpha\epsilon = \mp\frac{4r^2}{(2\xi\xi')^4}\left(-\frac{3}{\xi^2}\frac{u}{(u^2+1)^{7/2}} + \frac{1}{\xi\xi'}\frac{1-2u^2}{(u^2+1)^{7/2}}\right) \tag{90}$$

We now take advantage of the fact that the integral, Eq.(87), is independent of the synchronous time ξ. This simplifies the evaluation of the integral considerably because one may assume

$$1 \ll \frac{\xi}{\xi'}$$

without changing the value of the integral. The final outcome is that (i) in each of the expressions, Eqs.(88)-(90), only the last term contributes to the r-integral and (ii) the integral assumes a simple mathematical form if one introduces

$$u = \frac{\xi^2 - \xi'^2 - r^2}{2\xi\xi'}, \qquad du = -\frac{2r\, dr}{2\xi\xi'}$$

as the new integration variable. With this scheme one has

$$\int_0^\infty \cdots \frac{r^2 r\, dr}{(2\xi\xi')^2} = \frac{1}{2}\int_{(\xi^2-\xi'^2)/2\xi\xi'}^{-\infty} \cdots \left(\frac{\xi}{2\xi'} - \frac{\xi'}{2\xi} - u\right)(-)du \tag{91}$$

The to-be-used integrands have the form

$$\frac{1}{(u^2+1)^{n/2}}, \quad \frac{u}{(u^2+1)^{n/2}} \qquad\qquad n = 5, 7, \cdots ,$$

both of which are always less than one in absolute value, even when they get multiplied by u. Consequently, one is perfectly justified in saying that

$$\int_0^\infty \cdots \frac{r^2 r\, dr}{(2\xi\xi')^2} \to \frac{\xi}{4\xi'}\int_{-\infty}^{\infty} \cdots du \quad \text{whenever} \quad \xi' \ll \xi \; . \tag{92}$$

Taking note that only the last terms of Eqs.(88)-(90) give nonzero contribution, apply the limiting form, Eq.(92), to evaluate the integral, Eq.(87). One finds that

$$\begin{aligned}\int_{-\infty}^{\infty}\int_0^\infty\int_0^{2\pi} T^{\xi}_{\ \tau}\, \xi d\tau\, r dr\, d\theta &= \frac{(2\pi a^2)^2 2\pi}{4\pi}\frac{\pm 4}{(2\xi')^2}\frac{1}{4\xi'^2}\int_{-\infty}^{\infty}\left(\dddot{q}^2\int_{-\infty}^{\infty}\frac{du}{(u^2+1)^{5/2}} + \ddot{q}^2\int_{-\infty}^{\infty}\frac{du}{(u^2+1)^{5/2}}\right)d\tau \\ &= (\pi a^2)^2\frac{\pm 1}{2\xi'^4}\int_{-\infty}^{\infty}\left(\dddot{q}^2\int_{-\pi/2}^{\pi/2}\cos^3\phi\; d\phi + \ddot{q}^2\int_{-\pi/2}^{\pi/2}\cos^3\phi\; d\phi\right)d\tau\end{aligned} \tag{93}$$

The value of the integral

$$\int_{-\pi/2}^{\pi/2}\cos^3\phi\; d\phi = \frac{4}{3}$$

implies that the final result is

$$\int_{-\infty}^{\infty}\int_0^\infty\int_0^{2\pi} T^{\xi}_{\ \tau}\, \xi d\tau\, r dr\, d\theta = (\pm)\frac{(\pi a^2)^2}{\xi'^4}\int_{-\infty}^{\infty}\left(\frac{2}{3}\,\dddot{q}^2 + \frac{2}{3}\ddot{q}^2\right)d\tau \; , \tag{94}$$

the total momentum into the τ-direction radiated by a magnetic dipole accelerated in Rindler sector I (upper sign) or in Rindler sector II (lower sign). This is the result stated by Eq.(66)

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[1] Section V in A. Einstein, Jahrbuch der Radioaktivitaet und Electronik 4, p.411 (1907); English translation in *The Collected Papers of Albert Einstein, Vol. 2, The Swiss Years: 1900-1909*, translated by Anna Beck, Peter Havas, Consultant, (Princeton University Press, Princeton, N. J., 1989) p 252

[2] Instead of meter sticks with equally spaced markings, it is better to have equally spaced detectors and probes. This means that instead of measuring a particle in relation to the meterstick markings, measure it by the "click" this particle produces in the corresponding detector. Furthermore, by having the clocks keep track of the relative phase between adjacent detection probes, one can measure the wavelength of a electromagnetic wave train passing through the lattice work of equally spaced probes.

[3] The other breakthrough was, of course, his focus on the "principle of the uniqueness of free fall" ("Eotvos property"), which he identified as a manifestation of his "equivalence hypothesis" according to which a body's acceleration in a static gravitational field is merely a kinematical effect due to accelerated reference frame relative to which the body is observed.

[4] U.H. Gerlach, Phys. Rev. D **59**, p.104009 (1999) or gr-qc/9911016 ; See also U.H. Gerlach in *Proc. 8th Marcel Grossmann Meeting*, edited by T. Piran (World Scientific, Singapore, 1999), 952-954 or gr-qc/9910114

[5] See, for example, section 71 in L.L. Landau and E.M. Lifshits, *The Classical Theory of Fields* (Addison-Wesley Publishing Co., Inc, Reading, 1962)

[6] See, for example, Section 1.2, including Figure 1.4, in C.W. Misner, K.S. Thorne, and J.A. Wheeler, *Gravitation* (W.H. Freeman and Co., San Francisco, 1973)

[7] There are additional difficulties which are spelled out in C.W. Misner, K.S. Thorne, and J.A. Wheeler, *Gravitation* (W.H. Freeman and Co., San Francisco, 1973), Sec. 6.3, p.168

[8] The concept of a *recording* clock was first made explicit by Taylor and Wheeler in Sections 2.4 of *SPACETIME PHYSICS*, (W.H. Freeman and Co., San Francisco, 1966) and in Sections 2.6 and 2.7 of *SPACETIME PHYSICS, 2nd Edition*, (W.H. Freeman and Co., San Francisco, 1992). Unlike theirs, each of our recording clocks resembles the one found in a GPS satellite: each one also includes a transmitter.

[9] See, e.g., M. Born and E. Wolf, *Principles of Optics*, sixth edition, (Pergamon Press, Elmsford, N.Y., 1980), p.312

[10] *Notation:* (i) Throughout Section III and from Section V onward *primed* coordinates, such as $(\tau', \xi', r', \theta')$ or (t', z', r', θ'), refer to the source (transmitter) in Rindler sector I and/or II, while *unprimed* coordinates such as (ξ, τ, r, θ) or (t, z, r, θ), refer to the observer(receiver) in Rindler sector F. Also note that in listing these coordinates, e.g. (τ', ξ') or (ξ, τ), the *timelike coordinate is first*, followed by the spacelike coordinate(s).

[11] Remark by E. Wigner on page 285 in the discussion following papers by S.S. Chern and T. Regge in *Some Strangeness in Proportion*, Edited by Harry Wolf (Addison-Wesley Publishing Company, Inc., Reading, Mass., 1980)

[12] E. Schrödinger, *EXPANDING UNIVERSES*, (Cambridge University Press, New York, 1956), p. 20

[13] The sourceless field equations in Rindler sector I have been considered by P. Candelas and D. Deutsch, Proc.E Roy.Soc.**A 354**,79 (1977) and by A.Higuchi, G.E.A. Matsas, and D. Sudarsky, Phys. Rev. **D 46**, 3450 (1992)

[14] F.J. Alexander and U.H. Gerlach, gr-qc/99100086 ; Phys. Rev. **D 44**, 3887 (1991) exhibit the inhomogeneous T.E. and T.M. wave equations in terms of gauge-invariant mode expansions.

[15] A discussion of Hertz's electric and Righi's magnetic 'super potential' vectors are given in a tutorial article "Classical Electrodynamics" by M. Phillips, page 74 in *Encyclopedia of Physics, Volume IV* edited by S. Flűgge (Springer-Verlag, Berlin, 1962). This tutorial also refers to Whittaker's result.

[16] Simply express G as a double Fourier integral. Using Cauchy's integral formula, evaluate the first Fourier (frequency) integral first. Do this by having the integration path go slightly above the two poles on the real frequency axis. The result is the sum of two contributions. This yields the sum of two Fourier integral expressions for the zero order Hankel function of the first and second kind respectively. Their sum is the zero order Bessel function.

[17] A. Sommerfeld, *Partial Differential Equations in Physics*, (Academic Press, New York, 1949), Sec. 21, p.108

[18] We have included a factor of two on the right hand side of Eq.(47). We do this in order to guarantee that when $m = 0$ and when one considers a circular loop of area πa^2 localized at $\xi' = \xi_{I,II}$, as in Eq.(60), then the $d\xi'$-integral of Eq.(47) yields the standard value $\int_0^\infty S_{I,II}(\tau', \xi') d\xi' = \pi a^2 \times (proper\ current)$ for the proper magnetic dipole moment.

[19] Remember that it is $-\delta^m_{II}$ which expresses a phase increase because in II τ increases towards the past.

[20] T. Padmanabhan, Phys. Rev. Lett. **64**, 2471 (1990)