- C^r diffeomorphism $f : M \to M$ is *Anosov* if there is an invariant splitting $TM = E^u \oplus E^s$ such that for an $a > 0$,

$$\|Df(v)\| \geq e^a \|v\|, v \in E^u,$$

$$\|Df^{-1}(v)\| \geq e^a \|v\|, v \in E^s.$$

- There are invariant foliations W^u and W^s tangent to E^u and E^s. Write d^u_x for induced Riemannian metric on leaf $W^u(x)$ through x.

- Define for $y, z \in W^u(x)$,

$$\beta_x(y,z) = \sup\{t \in \mathbb{R} : d^u_{f^t x}(f^t x, f^t y) \leq 1\}.$$

  (non-integer t: use the suspension of f).

- *Hamenstädt metric* on $W^u(x)$: $\rho^u_x(y,z) = e^{-a\beta_x(y,z)}$.

- *Scaling property*: $f_*\rho^u_x = e^a \rho^u_{f(x)}$.

- 3-dimensional Heisenberg group is given by

$$G = \left\{ \begin{pmatrix} 1 & x & z \\ 0 & 1 & y \\ 0 & 0 & 1 \end{pmatrix}, x, y, z \in \mathbb{R} \right\}.$$

  Equip with left-invariant Riemannian metric.

- Left-invariant contact distribution $H = \langle \partial_x, \partial_y + x\partial_z \rangle$ on G.

- *Carnot-Caratheodory metric* ρ_G on G: $\rho_G(p, q) = \inf \ell(\gamma)$, γ a C^1 curve *tangent to H* joining p to q ($\ell(\gamma)$ the Riemannian length of γ).

- Expanding automorphism $g(x, y, z) = (e^a x, e^a y, e^{2a} z)$ is *conformal* for ρ_G: $g_* \rho_G = e^a \rho_G$.

- Take $a > 0$ same as first slide.

Now consider an Anosov diffeomorphism f with $\dim E^u = \dim E^s = 3$. Write $f_x = f|_{W^u(x)}$.

- The expanding automorphism g is a *conformal unstable leaf model* for f if there exists a family $\{T_x\}_{x \in M}$ of L-biLipschitz homeomorphisms $T_x : (W^u(x), \rho^u_x) \to (G, \rho_G)$ such that $f_x = T_{f(x)}^{-1} \circ g \circ T_x$.

Question

Suppose that g is a conformal unstable leaf model for f and f^{-1}. Is there a lattice $\Gamma \subset G \times G$ such that f is C^r-conjugate to an algebraic automorphism of $G \times G / \Gamma$?