Conjecture: Uniformly hyperbolic actions of large groups are classifiable.

The actions: H Lie group $\Lambda < H$ cocompact

$f \in \mathrm{Aff}(H/\Lambda) \Rightarrow \exists$ A an automorphism of H and $g \in H$

e.g. $f \in SL_n\mathbb{Z}$ acting on $\mathbb{T}^n$

s.t. $f(h\Lambda) = g A(h\Lambda)$

note: need $A(\Lambda) = \Lambda$.

if C is compact group in $\mathrm{Aff}(H/\Lambda)$

define $G\mathrm{Aff}(C \backslash H/\Lambda) = Z_{\mathrm{Aff}(H/\Lambda)}(C)$

Γ a group $\rho: \Gamma \to G\mathrm{Aff}(C \backslash H/\Lambda)$

is called a generalized affine action.

If M is compact manifold can also look at skew product actions of Γ on $\underline{C\backslash H/\Lambda} \boxed{\times M}$

$\alpha: \Gamma \times C\backslash H/\Lambda \to \mathrm{Diff}(M)$ cocycle

called a skew product "generalized quasi-affine" if $\alpha: \Gamma \times C\backslash H/\Lambda \to \mathrm{Isom}(M,g) \subset \mathrm{Diff}(M)$

<u>Conjecture I:</u> Let G be a ½ simple Lie group of real rank ≥ 2 and $\Gamma \subset G$ irreducible lattice. Assume $\rho: \Gamma$ or $G \to \mathrm{Diff}^{\infty}(N)$ and that $\exists\, \gamma \in \Gamma$ s.t. $\rho(\gamma)$ has <u>dominated</u> <u>splitting</u> then ρ is generalized quasi-affine.

$\Gamma = SL_n(\mathbb{Z})$
$n \geq 3$.

N compact manifold.

Now let $\Gamma = \mathbb{Z}^k$ or $\mathbb{R}^k$ $k \geq 1$

$\rho : \Gamma \longrightarrow \mathrm{Diff}^{\infty}(N)$ N compact mfld

two assumptions 1) ρ has no "rank 1 factor"

ie $\exists$ $f : N \to X$ Γ equivariant s.t. f C^{∞}

and the Γ action on X factors through $\mathbb{Z}$ or $\mathbb{R}$.

2) $\exists$ $\gamma \in \Gamma$ s.t. $\rho(\gamma)$ has dominated splitting.

then

Conjecture II: ρ is a skew product

over a generalized affine action

and the fiber directions are "slow".

RK: Not true C^1 by Rodriguez Hertz - Wang