A Three Manifold carrying infinitely many Anosov flows

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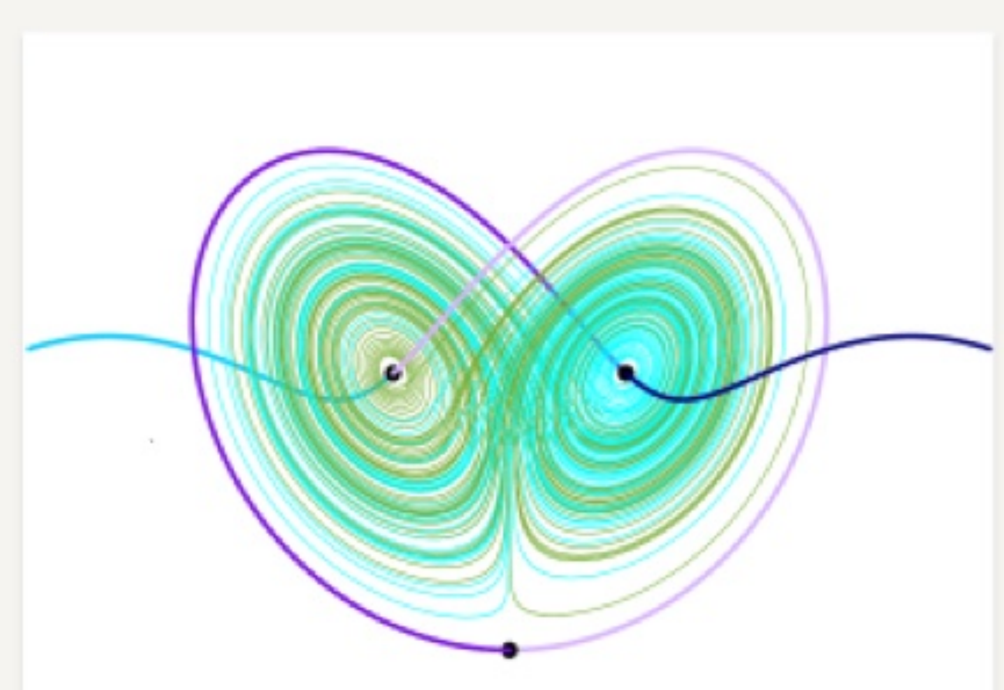
Joint work with Adam Clay

We wish to study Anosov flows on the trefoil knot complement $S^3 \setminus$ 

Motivation:

1. Existence of a trefoil knot in the Lorenz equations / A geometric Lorenz model, forcing the periodic orbits to be related to closed geodesics on the modular Surface.

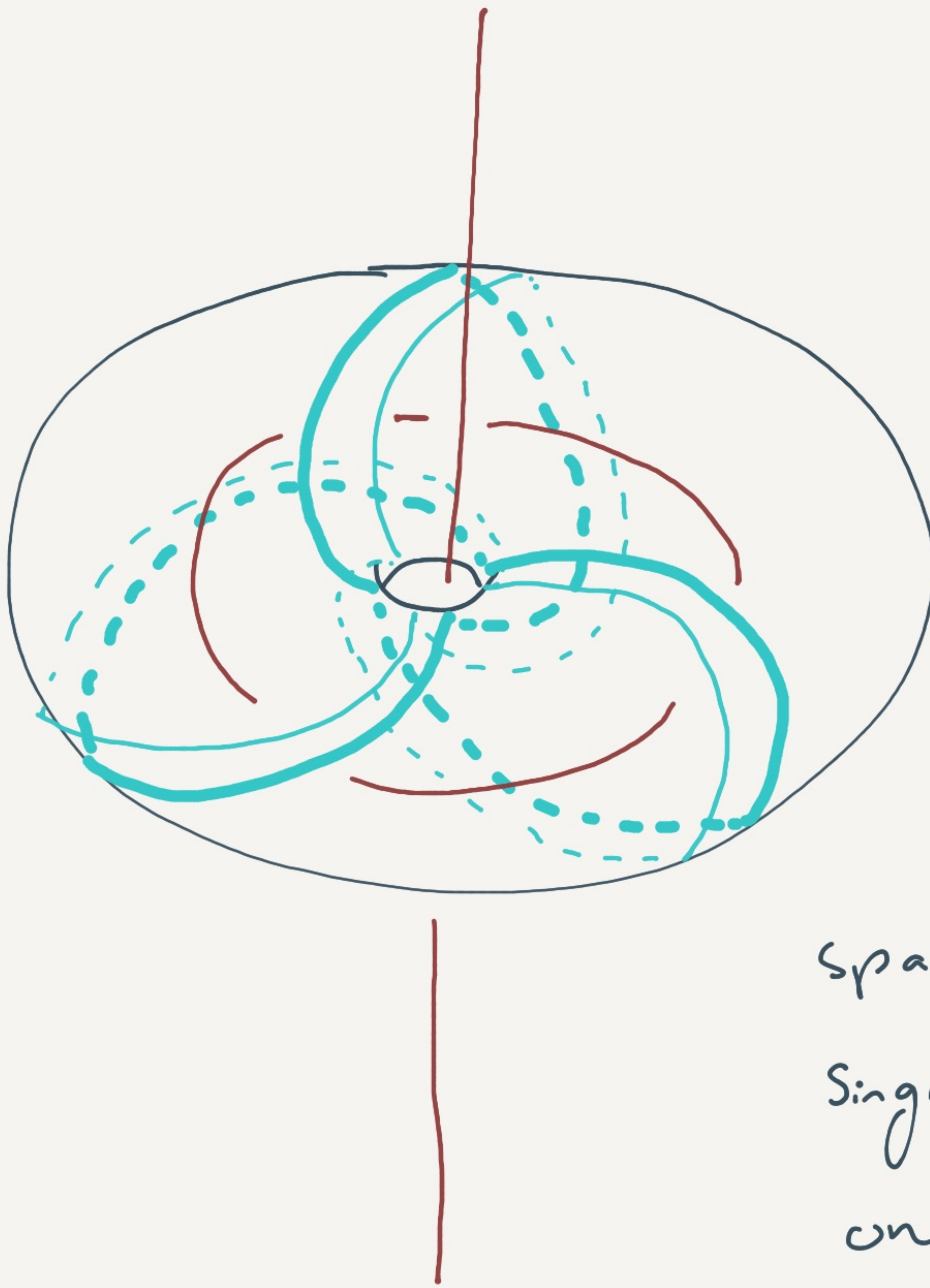


We wish to study Anosov flows on the trefoil knot complement $S^3 \setminus$ 

Motivation:

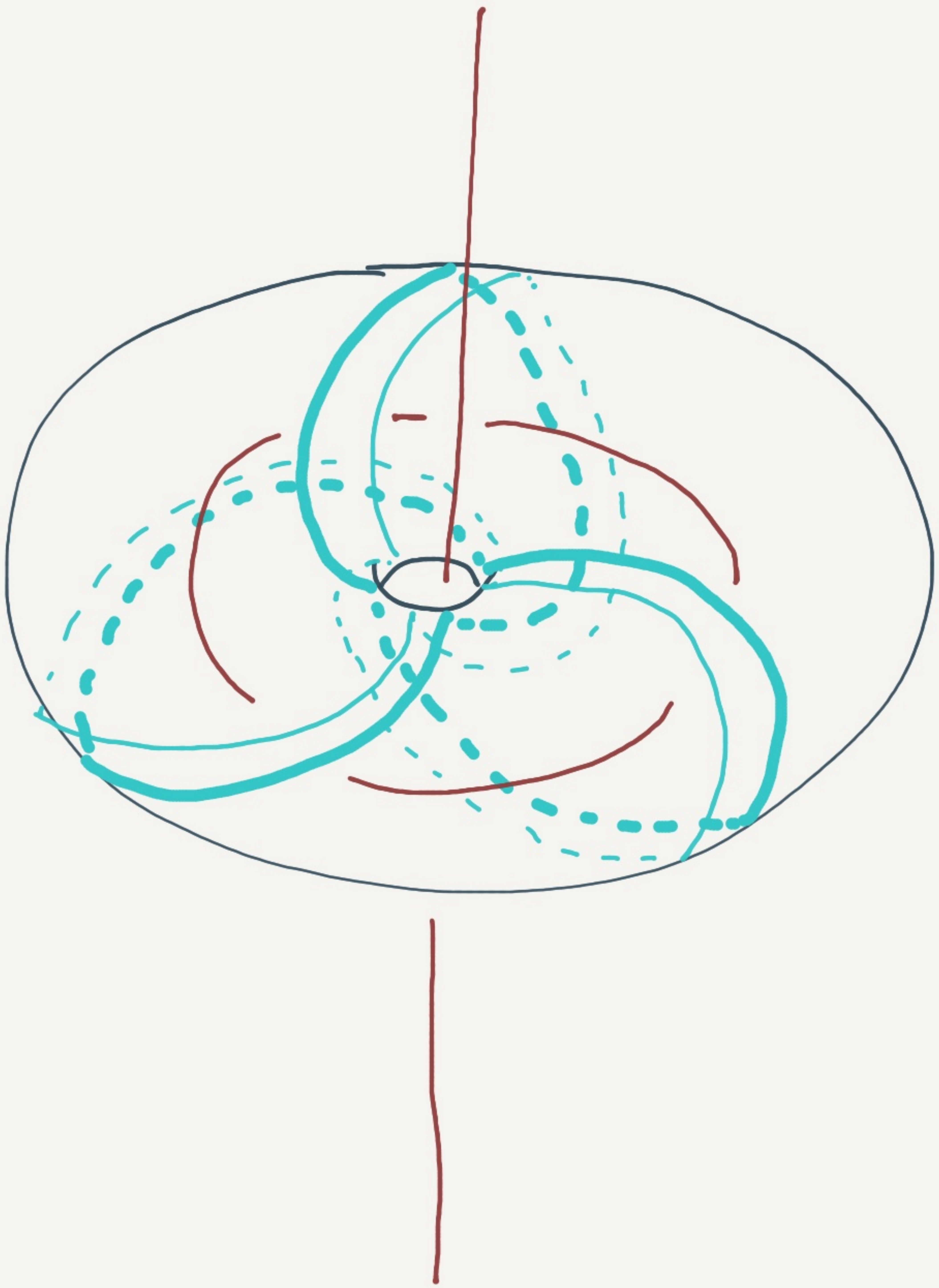
2. Can one glue trefoil complements and get a closed three manifold with ∞ly many Anosov flows?

The Trefoil Complement

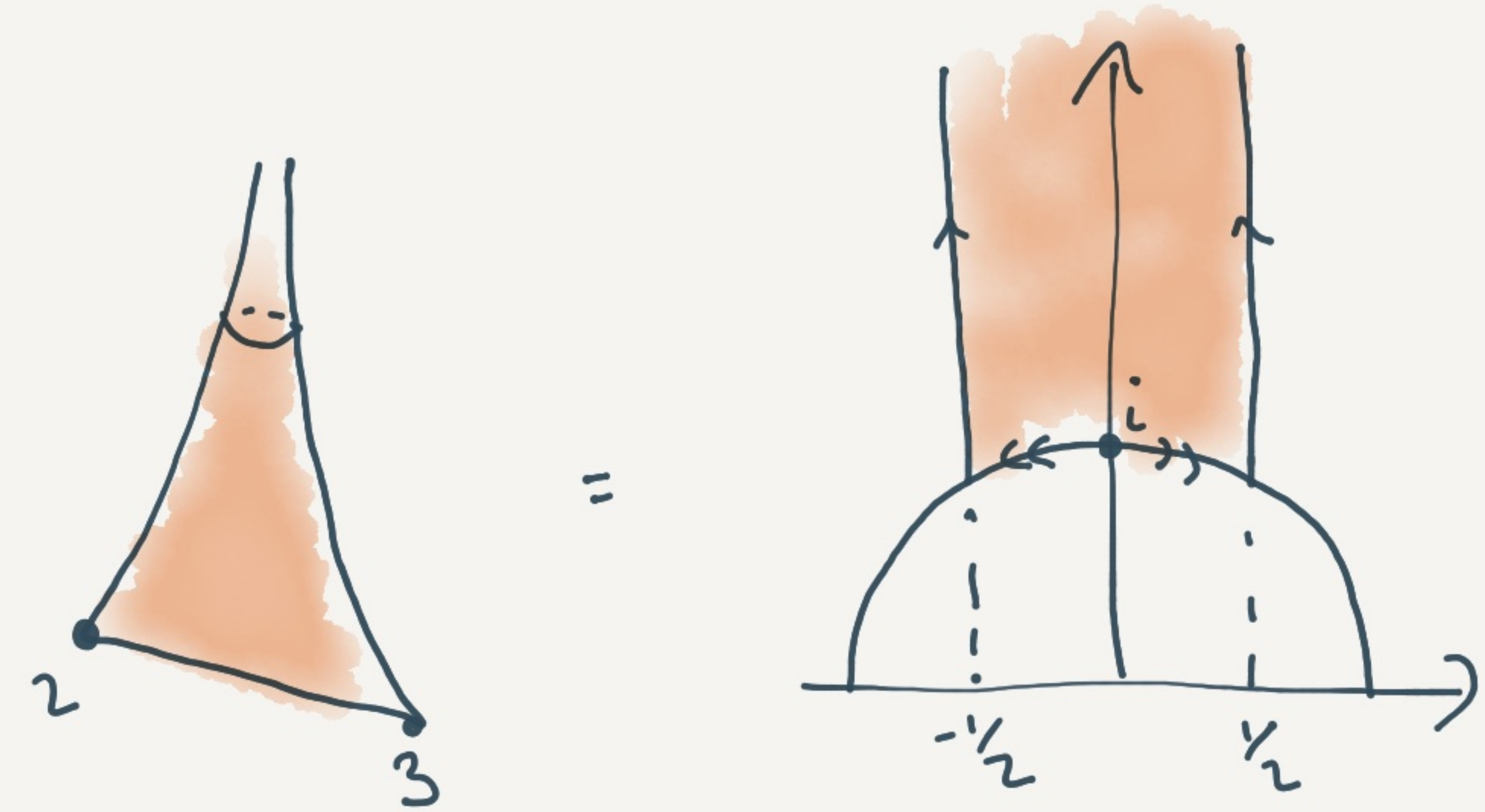


A Seifert fiber space with two singular fibers and one cusp

The Trefoil Complement

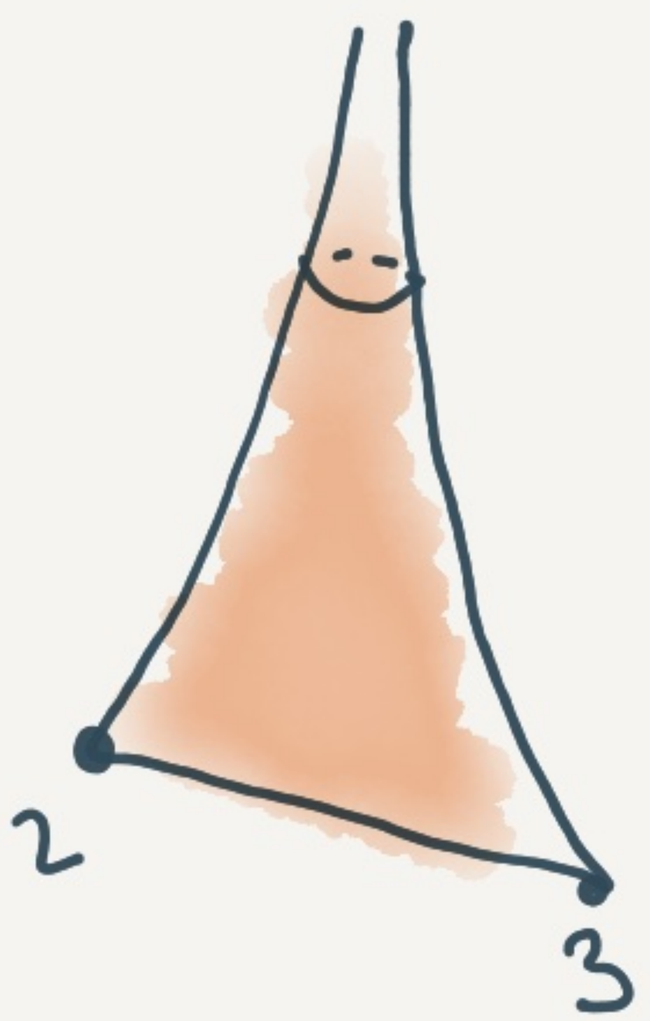


Its orbit space is the modular surface



$= \mathbb{H}^2 / PSL_2(\mathbb{Z})$

$$S^3 \setminus \quad = T^1(\mathrm{Mod}).$$

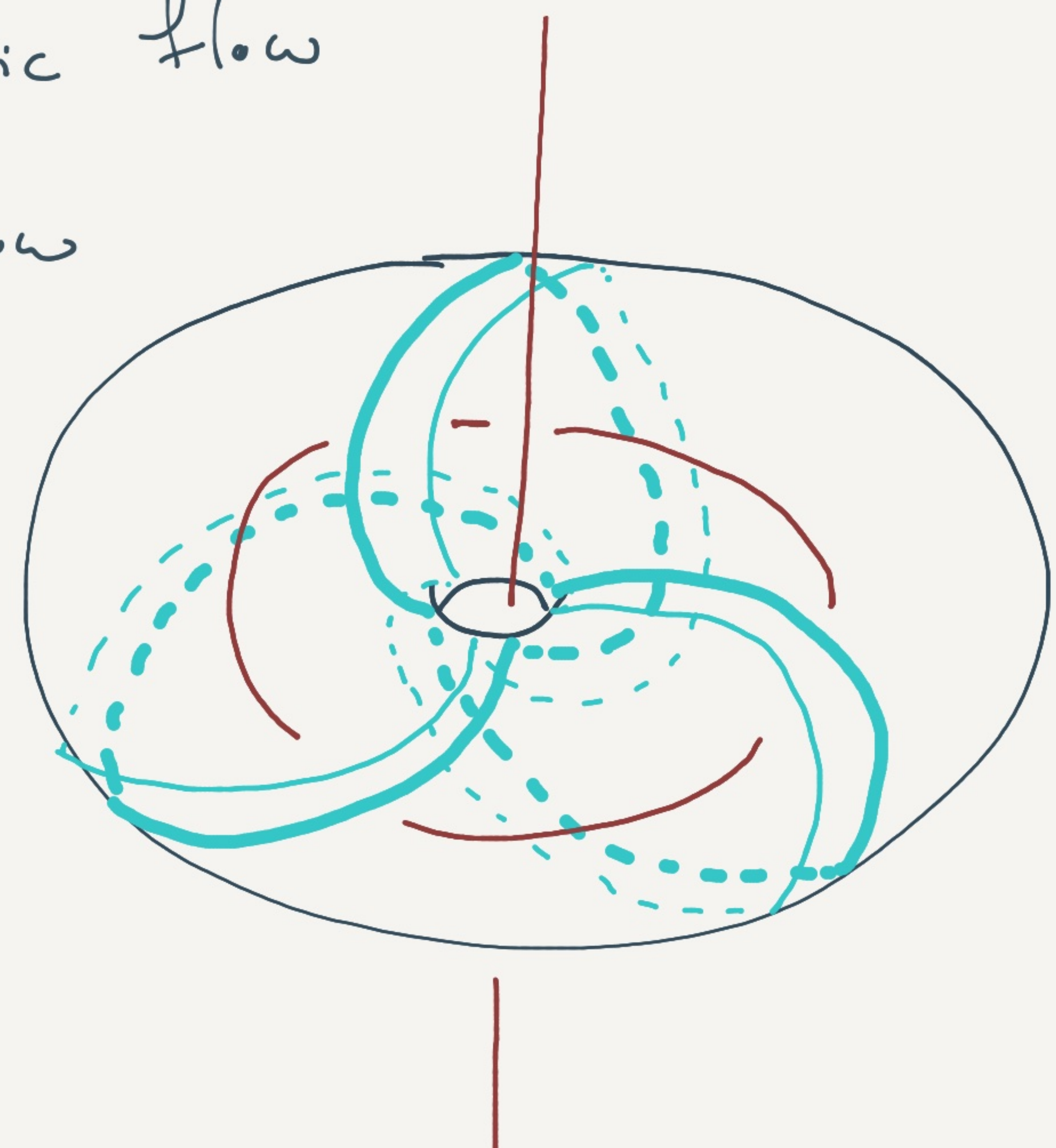


Mod is a hyperbolic surface,

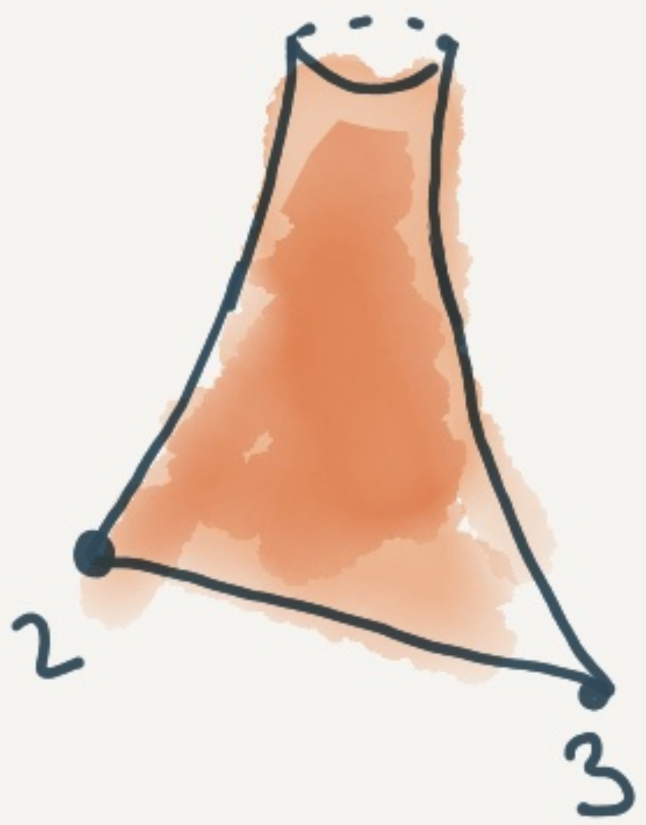
So its geodesic flow

is an Anosov flow

on $S^3 \setminus$

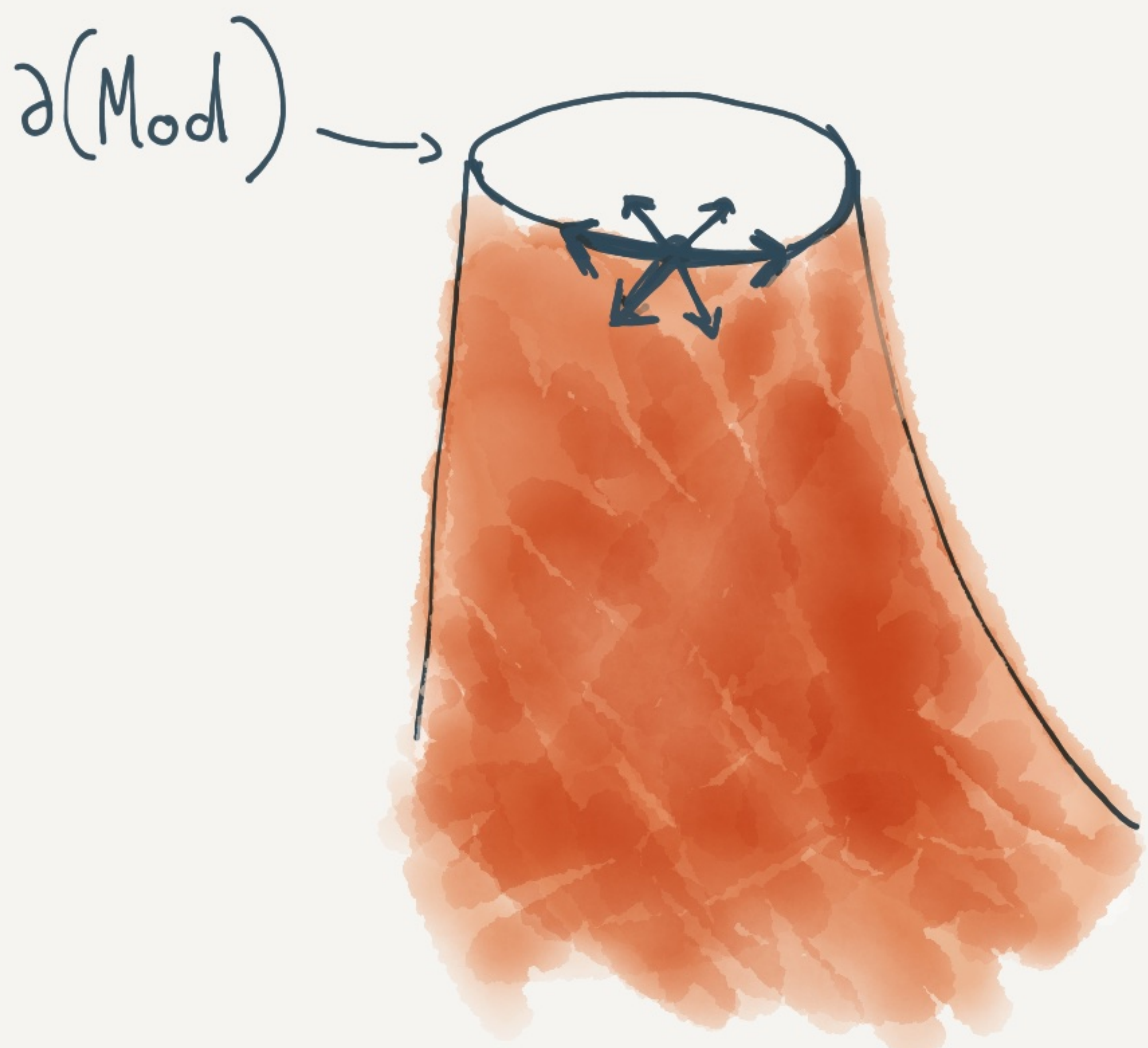


$M = S^3 \setminus$  $= T^1(\mathrm{Mod})$.



But it's better
to compactify!

The boundary ∂M is a Birkhoff torus with two tangent orbits:



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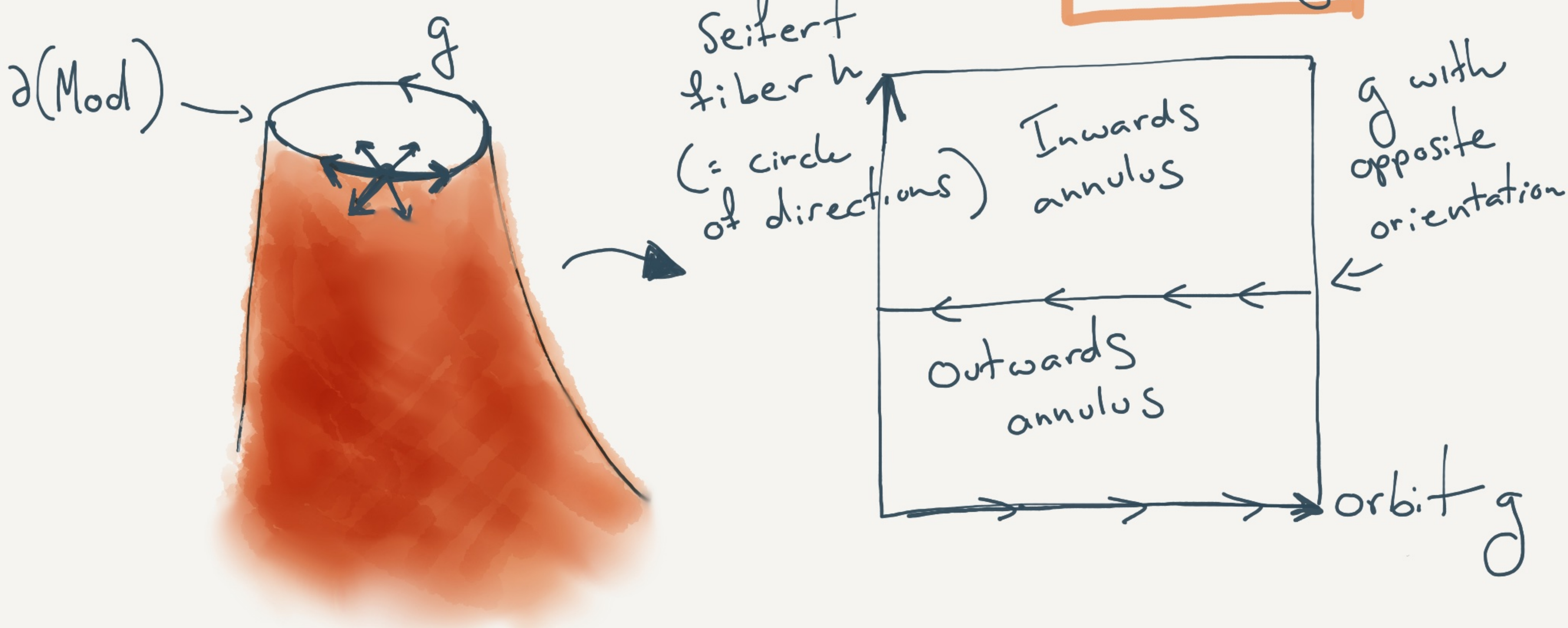
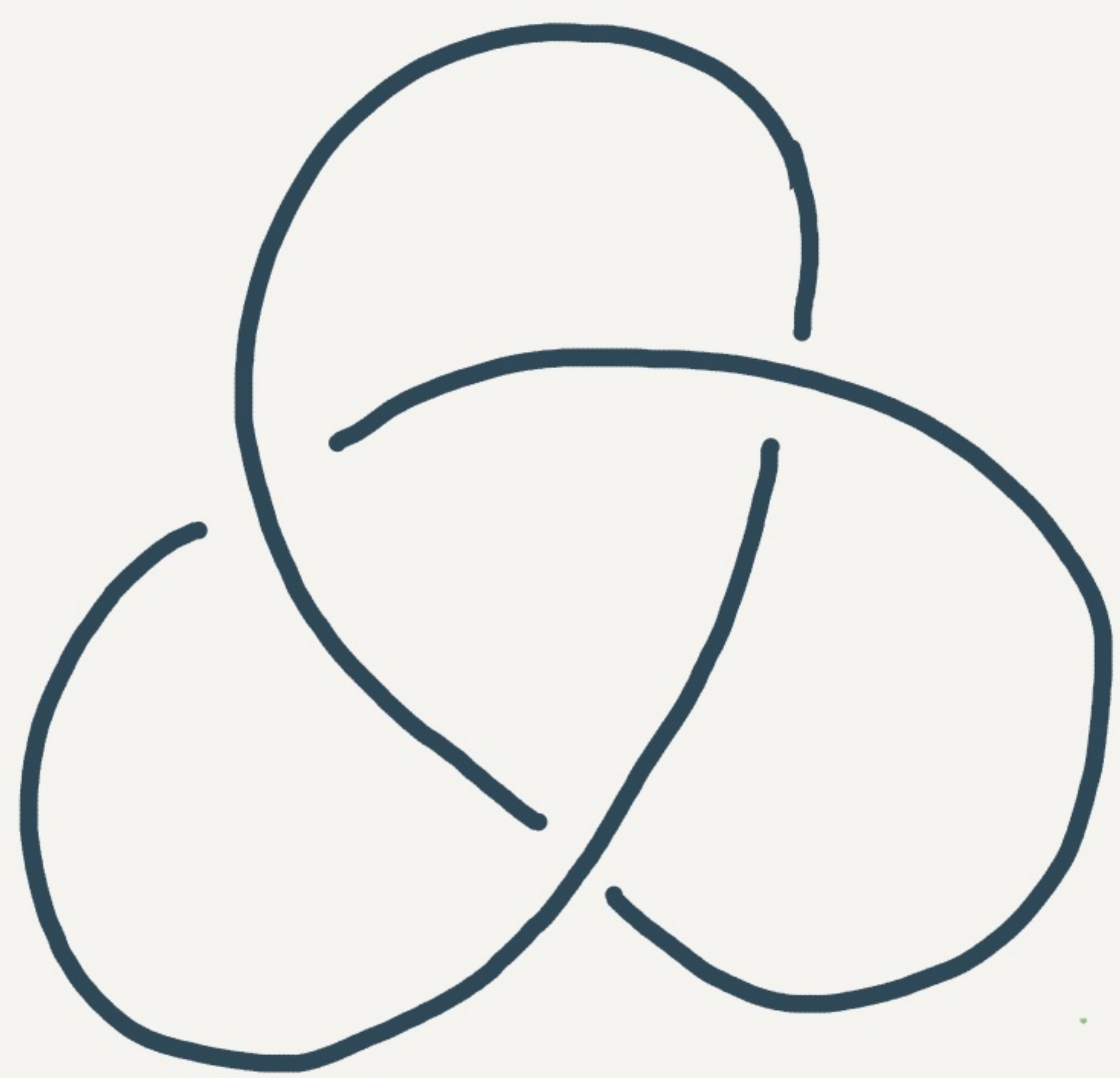
$$\boxed{\partial M = T^1 g}$$



There's another structure on $M = S^3 \setminus$  :

A Seifert surface T_0



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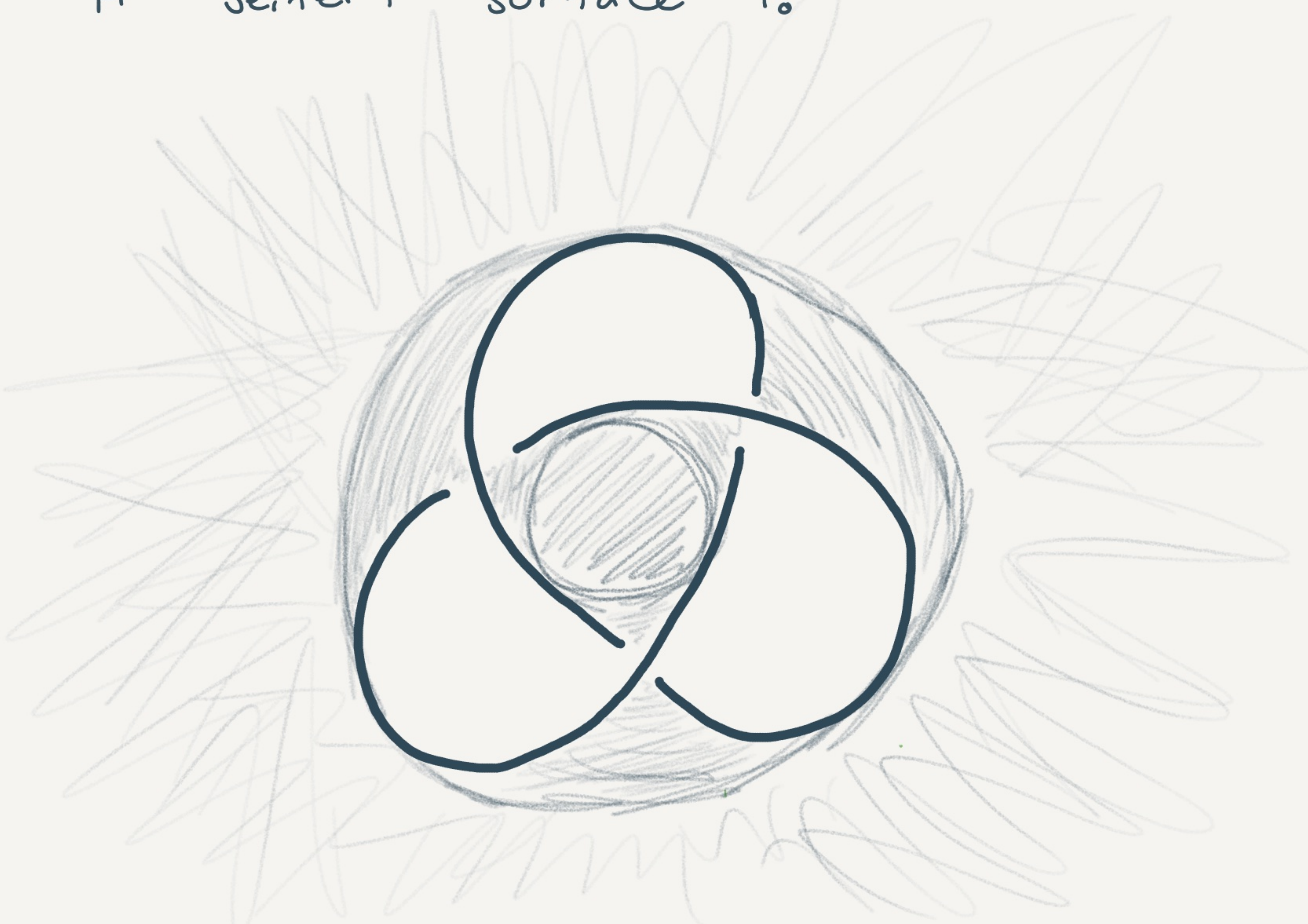

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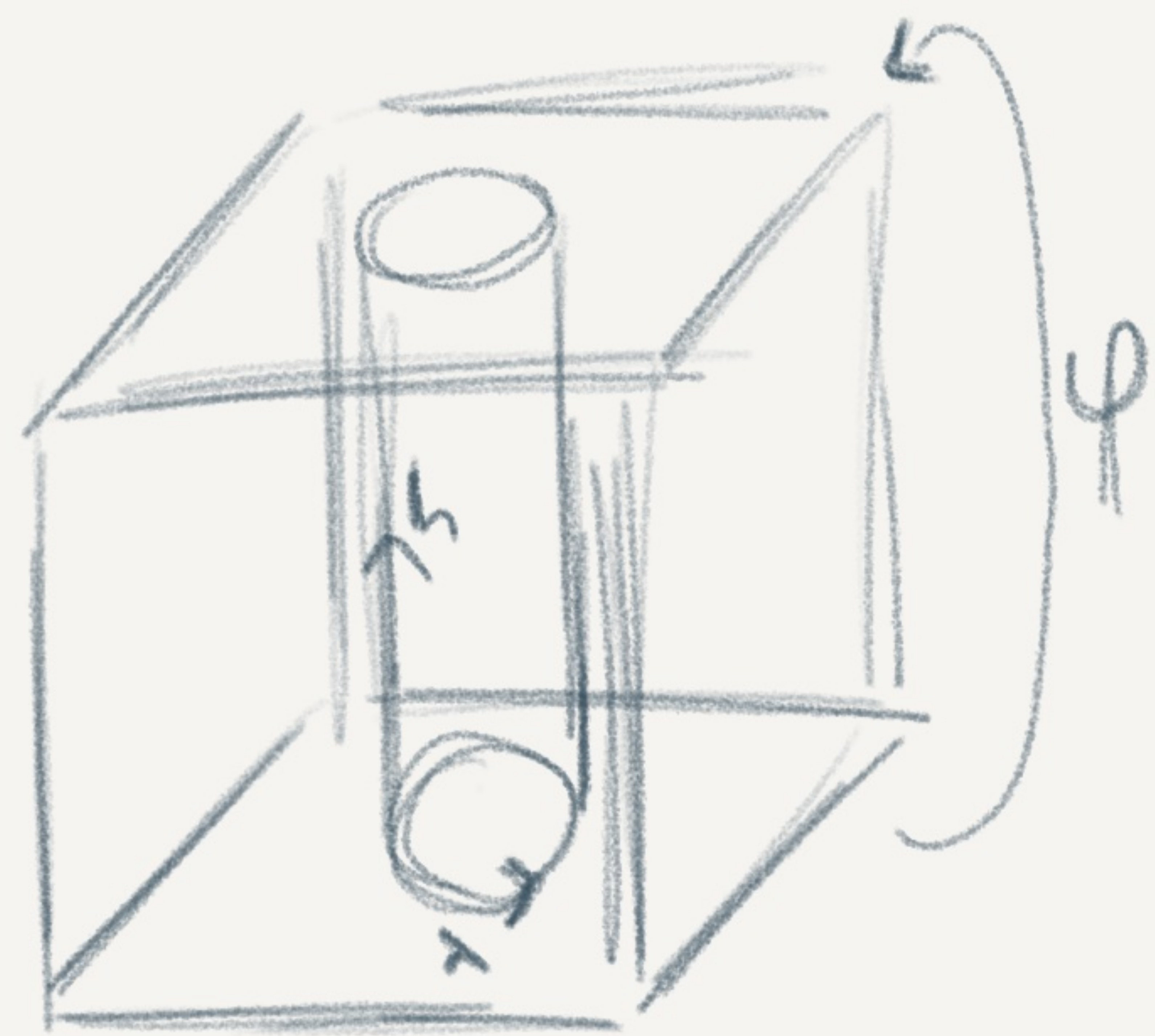
A Seifert surface T_0

Pushing T_0 along the fibers we get

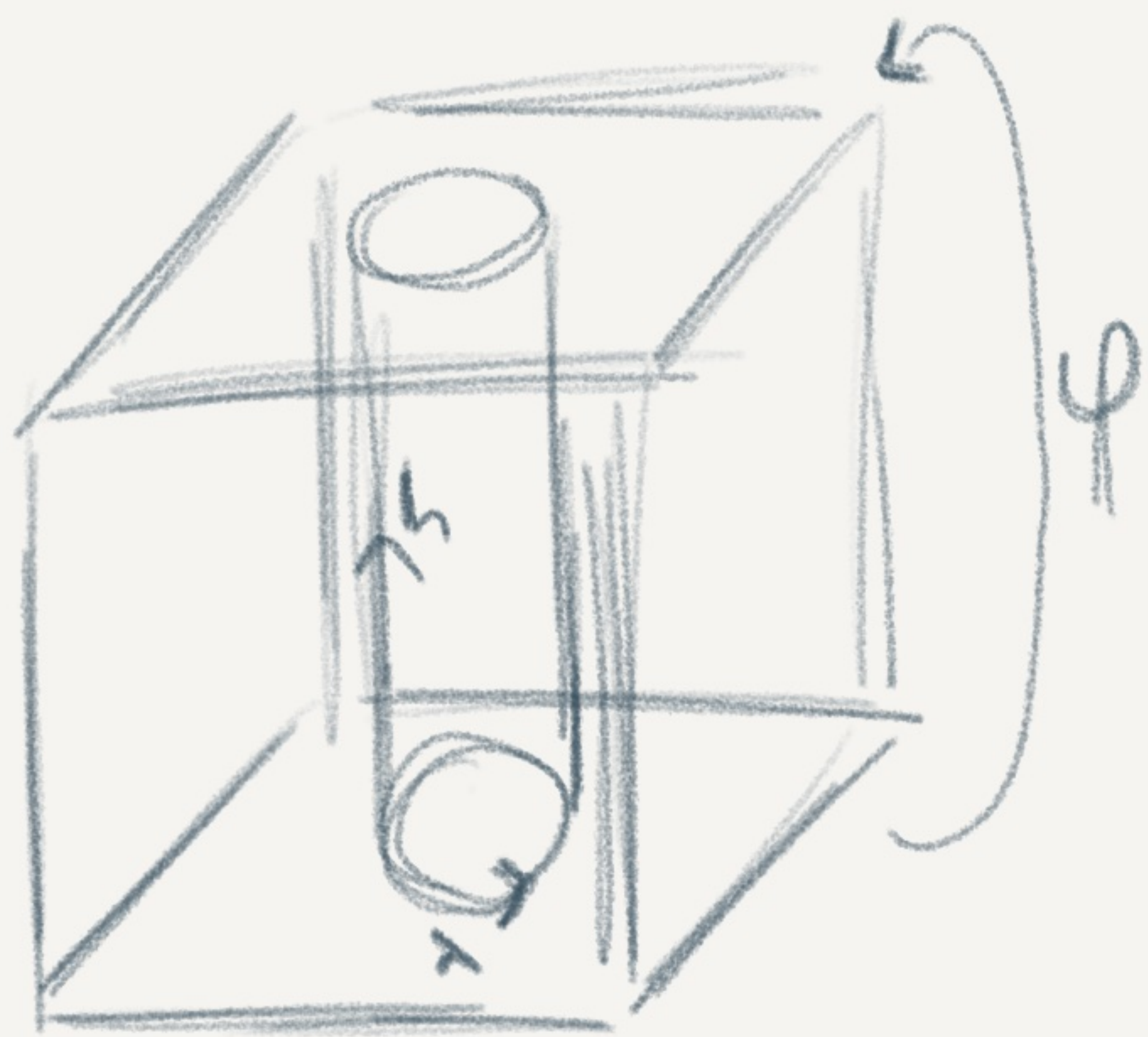
$\varphi : T_0 \to T_0$

$\varphi^6 = id.$

So M is a mapping cylinder with monodromy φ: $M = \dfrac{T_0 \times [0,1]}{(x,0) \sim (\varphi(x), 1)}$



And this allows us to construct covers $M \to M$.



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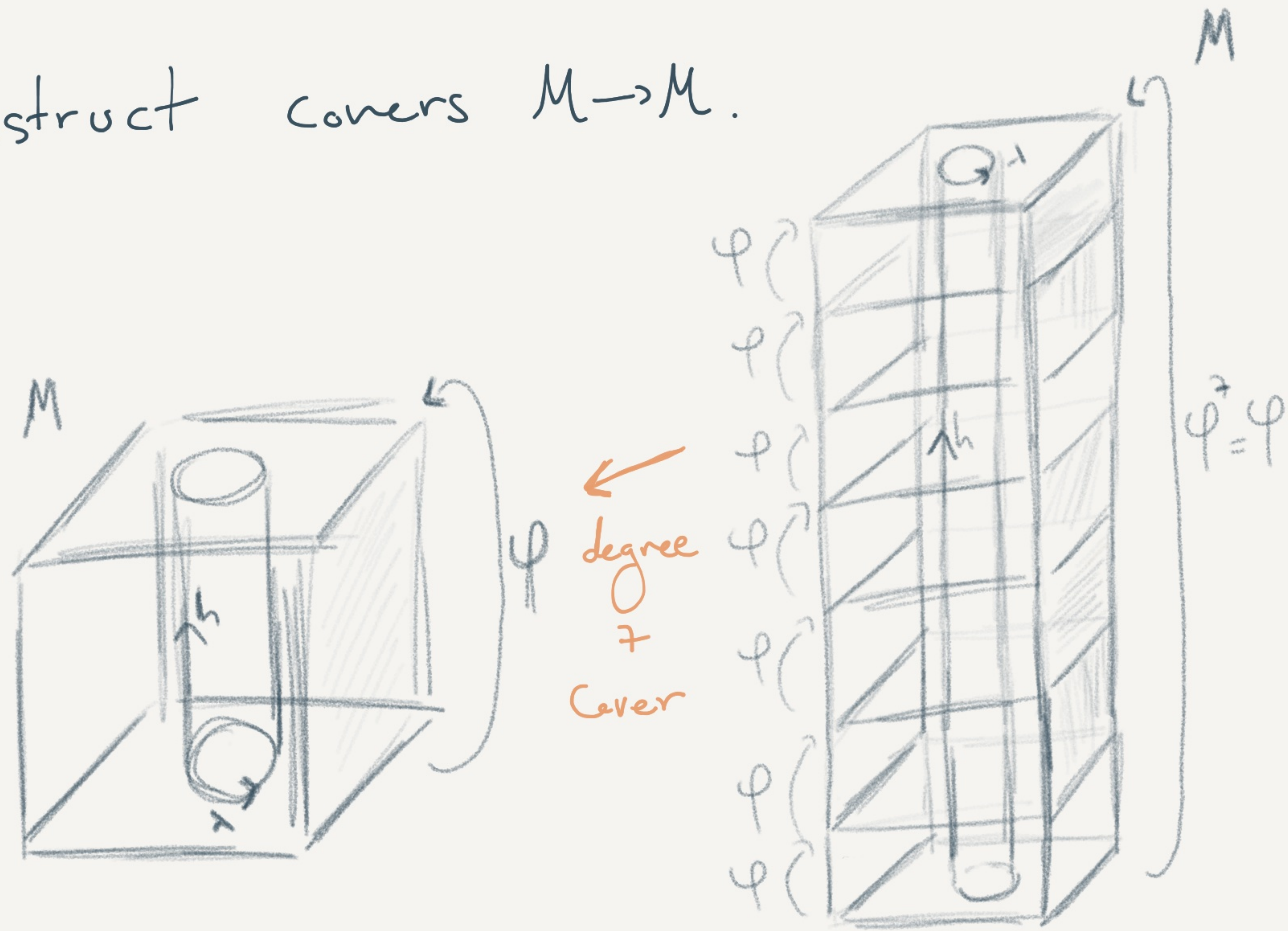

Similarly we can construct a cover $M \xrightarrow{d} M$ for degree $d = 1 + 6k$.

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$$\begin{tikzcd}
M \arrow[d, "p_d"'] & d[\lambda] \arrow[d] & [h] \arrow[d] & [\mu] + k[\lambda] \arrow[d] \\
M & [\lambda] & [h] & [\mu] = \text{periodic orbit}
\end{tikzcd}$$

$$\Big(\text{because } 6[\mu] = [\lambda] + [h] \Rightarrow p^{-1}(6[\mu]) = d[\lambda] + [h] = 6[\mu] + 6k[\lambda] \Big)$$

So by pulling back the geodesic flow Ψ_1:

Theorem: There exist infinitely many Anosov flows on $S^3 \setminus \mathcal{C}$, Ψ_d for any $d = 1 + 6k$, $k = 0, 1, 2, \ldots$.

Each has boundary a Birkhoff torus with two tangent orbits of homology $[\mu] + k[\lambda]$.

So can one glue these together and get a closed 3-manifold with ∞-ly many Anosov flows?

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We don't know! But maybe:

* There are other symmetries and self covers you can use.
* You need to find a map $\partial M \to \partial M$ matching orbits, and directions of Birkhoff annuli (for ∞ly many options at the same time.).