Partially hyperbolic maps with zero center Lyapunov exponents

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Goal: Relate 2 objects: Invariant foliations, invariant measures.

- $f : M \to M$ a Partially hyperbolic diffeomorphisms.

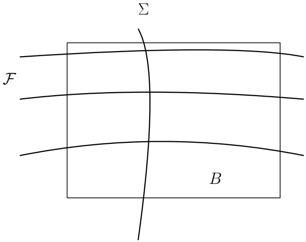
$$E^u \oplus E^c \oplus E^s$$

- $\mathcal{F}^u$ and $\mathcal{F}^s$ always exists.

- f is dynamically coherent if $\mathcal{F}^c$ exists.

- μ an f invariant measure.

Locally disintegrate μ with respect to $\mathcal{F}$.



$$\mu\mid_B = \int_\Sigma \mu^{\mathcal{F}} d\mu_\Sigma$$

Theorem (Avila-Viana-Wilkinson)

Let $\varphi_t : T^1S \to T^1S$ be a geodesic flow on a negative curvature surface, then there exists a C^1-neighborhood $\varphi_1 \in \mathcal{N} \subset Diff^2_{vol}(T^1S)$ such that for $f \in \mathcal{N}$ either

- *μ^c is atomic, or*
- *f is the time one map of an Anosov flow (μ^c is Lebesgue).*

Theorem (Hertz-Hertz-Tahzibi-Ures)

Let $f : M^3 \to M^3$ be a C^{1+} partially hyperbolic diffeomorphism with compact one dimensional center leaves and accessible, then either:

- *f has a unique m.m.e. with zero center Lyapunov exponent (μ^c is Lebesgue), or*
- *f has finite many m.m.e some with positive and some with negative exponents (μ^c is atomic).*

Theorem (Buzzi-Crovisier-P.-Tahzibi)

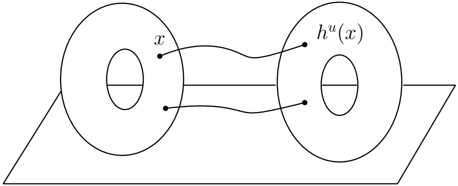
Let $\varphi_t : M^3 \to M^3$ be a transitive Anosov flow that is not a suspension, then there exists a C^1-neighborhood $\varphi_1 \in \mathcal{N} \subset Diff^2(M^3)$ such that for $f \in \mathcal{N}$ either

- *f has a unique m.m.e. and f is the time one map of an topological Anosov flow, or*
- *f has exactly 2 hyperbolic m.m.e. one with $\lambda^c > 0$ and one with $\lambda^c < 0$.*

Analyze $\lambda(x, v) = \lim_{n\to\infty} \frac{1}{n} \log \|Df^n(x)v\| d\mu$

Theorem (Avila-Viana Invariance Principle)

$f : M \to M$ be a partially hyperbolic skew product, then $\lambda(x, v) \le 0$ for $v \in E^c$ and μ a.e. $x \in M$ implies that μ^c is invariant under u-holonomies.



Corollary

If $\lambda^c = 0$ and μ projects on a measure with product structure then $x \mapsto \mu_x^c$ can be extended continuously to the support.

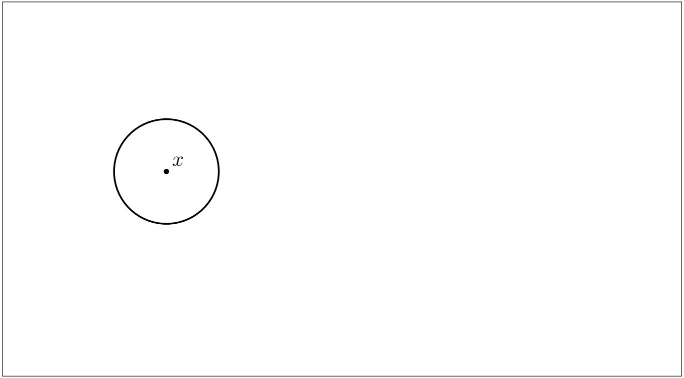
IMPORTANT:$\mathcal{F}^c$ compact foliation and Fiber bundle.

Quasi Isometric

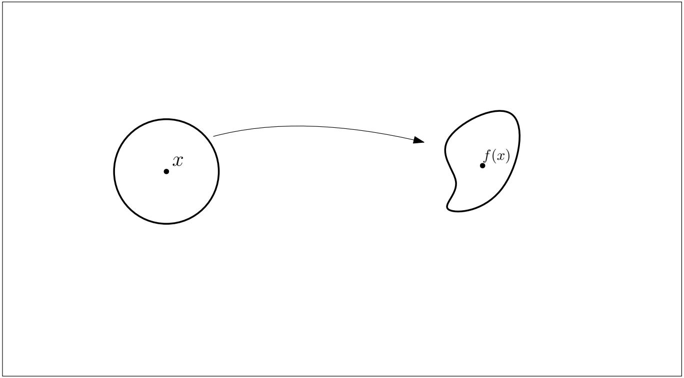
f is **quasi isometric on the center** if there exists $K, q > 0$ s.t for $n \geq 0$

$$K^{-1} d^c(x,y) - q \leq d^c(f^n(x), f^n(y)) \leq K d^c(x,y) + q.$$

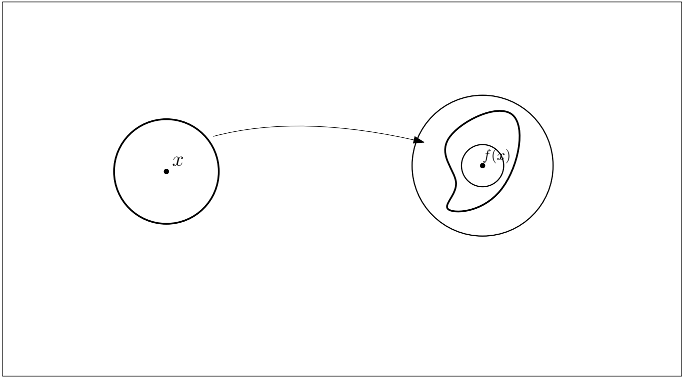
Quasi Isometric



Quasi Isometric



Quasi Isometric



Theorem (Crovisier-P.)

$f : M \to M$ a C^2 P.H. quasi isometric on the center, if $\lambda^c(x) \leq 0$ then μ^c is invariant by u-holonomies.

Corollary

If $\mu \sim \mu^{cu} \times \mu^{s}$ *and* $\lambda^{c} = 0$ *then* $x \mapsto \mu^{c}_{x}$ *is continuous on* $\mathrm{supp}(\mu)$.

Proof of BCPT: f close to φ_1.Buzzi-Fisher-Tahzibi proved that either: there are two, μ_+ and μ_- or all have zero exponents. Moreover if μ m.m.e. with $\lambda^c(\mu) \leq 0$ then μ^u is a Margulis family:

- μ^u_x is h^{cs} quasi invariant.
- μ is fully supported.

$\lambda^c(\mu) = 0$, $\mu \sim \mu^{cs} \times \mu^u$.

Proof of CP: