BEYOND UNIFORM HYPERBOLICITY 2025
MINI-COURSE ABSTRACTS

Fabrizio Bianchi (Università di Pisa, Italy) and Mary Yan He (University of Oklahoma, USA):

Thermodynamic formalism in holomorphic dynamics (1st week). We will first introduce the main concepts and questions of thermodynamic formalism in the setting of one-dimensional holomorphic dynamics. We will see how the complex setting often allows us to overcome the need for hyperbolicity assumptions usually made to study these problems in more general settings. We will then move to higher dimensions. Holomorphic dynamical systems in higher dimensions exhibit different behaviors from those in dimension 1, as holomorphic maps are not necessarily conformal anymore and several classical theorems in complex analysis no longer hold in several complex variables. We will introduce a volume dimension for measures with positive Lyapunov exponents as a dynamical replacement for the Hausdorff dimension and discuss its applications.

Jialun Li (École Polytechnique, France) and Wenyu Pan (University of Toronto, Canada):

Random walks on projective spaces (1st week) We consider the (semi)group action of $\mathrm{SL}_3(\mathbb{R})$ on $\mathbb{P}(\mathbb{R}^3)$, a primary example of non-conformal, non-linear, and non-strictly contracting action. The goal of these lectures is to present recent progress for studying the Hausdorff dimension of a dynamically defined limit set in $\mathbb{P}(\mathbb{R}^3)$, focusing on the result of generalizing the classical Patterson-Sullivan formula using the approach of stationary measures.

- Lecture 1: Introduction and statement of the main results (Pan)
- Lectures 2,3: Entropy growth argument for Bernoulli convolution based on [Hoc14]; non-concentration of stationary measures on arithmetic sequence via Fourier decay (Li)
- Lecture 4: Variational principle for Anosov representations; proof of the dimension formula of limit sets [LPX23] [JLPX23] (Pan)

References:

[Hoc14] Michael Hochman. *On self-similar sets with overlaps and inverse theorems for entropy.* Annals of Mathematics, 180(2):773–822, 2014.

[JLPX23] Yuxiang Jiao, Jialun Li, Wenyu Pan, and Disheng Xu. *On the dimension of limit sets on $\mathbb{P}(\mathbb{R}^3)$ via stationary measures: Variational principles and applications*, to appear in IMRN (arXiv:2311.10262), 2023.

[LL23] François Ledrappier and Pablo Lessa. *Exact dimension of Furstenberg measures.* GAFA, 33(1):245–298, 2023.

[LPX23] Jialun Li, Wenyu Pan, and Disheng Xu. *On the dimension of limit sets on $\mathbb{P}(\mathbb{R}^3)$ via stationary measures: The theory and applications*, arXiv:2311.10265, 2023.

[Rap21] Ariel Rapaport. *Exact dimensionality and Ledrappier-Young formula for the Furstenberg measure.* Trans. AMS, 374(7):5225–5268, 2021.

Kurt Vinhage (University of Utah, USA):
Rigidity of abelian actions (1st week) The talks include ideas from works with D. Damjanovic, R. Spatzier, A. Uzman, and D. Xu in various combinations.

- **Talk 1: Fundamentals of (partially) hyperbolic abelian group actions** — Revisiting core ideas in (partially) hyperbolic dynamics for abelian group actions, highlighting strengths and limitations in higher rank settings.
- **Talk 2: Tools for rigidity** — Describing tools for building homogeneous structures from hyperbolic actions, connecting with accessibility, the invariance principle, and Brin-Pesin bundle extensions.
- **Talk 3: Non-rigidity and partial rigidity** — Exploring settings where rigidity fails, including time changes and mixed rigid/non-rigid behavior even in presence of rank one factors.

Aaron Brown (Northwestern University, USA) and Federico Rodriguez Hertz (Penn State, USA):
Rigidity for generalized u-Gibbs states (2nd week) This course explores rigidity of u-Gibbs measures and the techniques involved in proving such results.

- u-Gibbs measures: random dynamics, stationary measures, skew extensions, u-invariance; $SL(2,\mathbb{R})$-actions and P-invariant measures; uu-Gibbs for flows and maps with dominated splitting.
- Homogeneous structures: normal forms; disintegration along measurable partitions; locally finite measures along unstable manifolds; homogeneity.
- Extra invariance of measures: standard and non-conformal dynamics adaptations.
- Ideas from the Brown–Eskin–Filip–Rodriguez Hertz proof: generalized u-Gibbs space, induced cocycle, entropy estimates.

Jérôme Buzzi (Université Paris-Saclay, France) and Ali Tahzibi (University of São Paulo, Brazil):
Unstable entropy in smooth ergodic theory (2nd week) Focusing on dynamics within unstable foliations, especially strongly unstable ones in partially hyperbolic diffeomorphisms, we connect these to expanding map dynamics and properties of SRB and MME measures.

- Definitions of measured/topological entropies for expanding foliations; variational principles.
- Identification of conditional measures and u-states; the invariance principle.
- Applications to partial hyperbolic dynamics: SRB measures for mostly contracting/expanding systems; MME for perturbations of Anosov flow time-one maps.

Dominik Kwietniak (Jagiellonian University, Poland):
Complexity of classification of dynamical systems (2nd week) This course examines how complex classification problems in dynamics can be, particularly when trying to determine conjugacy or isomorphism classes of systems. We use tools from descriptive set theory.

- **Lecture 1: The Toolbox** — Survey of the theory of complexity of equivalence relations.
- **Lecture 2: The Scoreboard** — Examples of classification and anti-classification in dynamics and ergodic theory.
- **Lecture 3: The Action** — Detailed proofs of anti-classification results.

Core References:

- Buzzi et al., *Open questions in descriptive set theory and dynamical systems*, arXiv:2305.00248.
- Deka et al., *Bowen's Problem 32 and the conjugacy problem for systems with specification.*
- Foreman, *What is a Borel reduction?*, Notices AMS 65(10):1263–1268, 2018.
- Foreman, *The complexity and the structure and classification of dynamical systems*, Springer, 2023.
- Hjorth, *Borel Equivalence Relations*, in Handbook of Set Theory, Springer, 2010.