CENTRALIZERS OF PARTIALLY HYPERBOLIC DIFFEOMORPHISMS IN DIMENSION 3

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Abstract. In this note we describe centralizers of volume preserving partially hyperbolic diffeomorphisms which are homotopic to identity on Seifert fibered and hyperbolic 3-manifolds. Our proof follows the strategy of Damjanovic, Wilkinson and Xu [DWX19] who recently classified the centralizer for perturbations of time-1 maps of geodesic flows in negative curvature. We strongly rely on recent classification results in dimension 3 established in [BFFP19].

In [DWX19], Damjanovic, Wilkinson and Xu investigate centralizers of certain partially hyperbolic diffeomorphisms and prove the following beautiful rigidity result: The centralizer of a perturbation of a time-1 map of an Anosov geodesic flow is either virtually $\mathbb{Z}$ or it is virtually $\mathbb{R}$. In the latter case the partially hyperbolic diffeomorphism is the time-1 map of a smooth Anosov flow.

The proof in [DWX19] works equally well in any dimension. Here we point out that, if one considers only 3-manifolds, then some lemmas can be strengthened to obtain the rigidity result for a much broader class of partially hyperbolic diffeomorphisms.

For any diffeomorphism $f\colon M \to M$, we denote the centralizer of f by

$$\mathcal{Z}(f) := \{g \in \mathrm{Diff}(M) \mid g \circ f = f \circ g\},$$

where $\mathrm{Diff}(M)$ is the space of C^1-diffeomorphisms of M.

We say that $f\colon M \to M$ is a *discretized Anosov flow* if f is a partially hyperbolic diffeomorphism such that there exists a (topological) Anosov flow $\varphi^t\colon M \to M$ and a function $h\colon M \to \mathbb{R}^+$ such that $f(x) = \varphi^{h(x)}(x)$ for all $x \in M$[1].

In this note, the partially hyperbolic diffeomorphism f is always assumed to be a C^∞ diffeomorphism.

Theorem A. *Let $f\colon M \to M$ be a volume-preserving partially hyperbolic diffeomorphism on a 3-manifold. If f is a discretized Anosov flow and $\pi_1(M)$ is not virtually solvable then either $\mathcal{Z}(f)$ is virtually $\{f^n \mid n \in \mathbb{Z}\}$ or f embeds into a smooth Anosov flow.*

Using the main results of [BFFP19], we then deduce the following results.

Theorem B. *Let $f\colon M \to M$ be a volume-preserving partially hyperbolic diffeomorphism on a Seifert 3-manifold which is homotopic to the identity. Then either $\mathcal{Z}(f)$ is virtually $\{f^n \mid n \in \mathbb{Z}\}$ or $\mathcal{Z}(f)$ is virtually $\mathbb{R}$ and a power of f embeds into an Anosov flow.*

Theorem C. *Let $f\colon M \to M$ be a volume-preserving dynamically coherent partially hyperbolic diffeomorphism on a hyperbolic 3-manifold. Then either $\mathcal{Z}(f)$ is virtually $\{f^n \mid n \in \mathbb{Z}\}$ or $\mathcal{Z}(f)$ is virtually $\mathbb{R}$ and a power of f embeds into an Anosov flow.*

Remark 0.1. Note that Theorem B is a generalization of the 3-dimensional case of Theorem 3 of [DWX19]. (One has to take a power of f to obtain the embedding into an Anosov flow only in the case when M is a k-cover of the unit tangent bundle of a hyperbolic surface or an orbifold, see Remark 7.4 in [BFFP19]).

Remark 0.2. The reason we exclude virtually solvable $\pi_1(M)$ in Theorem A is that, in this case, f would be a discretized Anosov flow of a suspension of an Anosov diffeomorphism. Thus f could fail to be accessible and the main motor of the proof, which is a dichotomy result by Avila, Viana

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[1] This is the same definition as in [BFFP19], see Appendix G of [BFFP19] for more details. Note that a discretized Anosov flow is a much broader class than what is called a discretized flow in [DWX19], which is just a time-1 map of an Anosov flow.

and Wilkinson [AVW15, AVW19], does not work. If one asks for f to be accessible, then Theorem A will apply even on manifold with virtually solvable fundamental group.

In particular, any dynamically coherent, accessible, volume-preserving partially hyperbolic diffeomorphism f on a manifold with virtually solvable fundamental group has centralizer virtually $\mathbb{Z}$ or virtually $\mathbb{R}$ in which case a power of f embeds into an Anosov flow. (The proof follows as in section 2, but using the classification results of Hammerlindl and Potrie (see [HP18]) instead of [BFFP19]).

As this note heavily relies on the arguments of [DWX19] to obtain Theorem A, we did not try to make it self-contained and refer to [DWX19] whenever an argument does not need substantial change.

1. Proof of Theorem A

Overall the proof follows the scheme of the proof of Theorem 3 of [DWX19]. The difference is in the following lemmas which are more general (when considering the 3-dimensional case) from their counterparts in [DWX19].

For $f\colon M \to M$ a dynamically coherent partially hyperbolic diffeomorphism, we denote by $\mathcal{W}^s$, $\mathcal{W}^u$, $\mathcal{W}^{cs}$, $\mathcal{W}^{cu}$, and $\mathcal{W}^c$ the stable, unstable, center stable, center unstable and center foliations of f, respectively. Recall that the foliations $\mathcal{W}^s$ and $\mathcal{W}^u$ are unique, but, in general, the others are not. Thankfully, for discretized Anosov flow, they are unique.

Lemma 1.1. *Let $f\colon M \to M$ be a discretized Anosov flow. Then there exists a unique pair of center stable $\mathcal{W}^{cs}$ and center unstable $\mathcal{W}^{cu}$ foliations that are preserved by f. Hence $\mathcal{W}^c$ is also unique.*

Proof. Since f is a discretized Anosov flow, it admits a pair of center stable and center unstable foliations such that a good lift $\widetilde{f}$ of f to the universal cover $\widetilde{M}$ fixes each leaf of the lifted foliations (see [BFFP19, Proposition G.1]). Thus, by [BFFP19, Lemma 12.6], these foliations are unique. □

As a direct consequence of Lemma 1.1, we obtain that, if $g \in \mathcal{Z}(f)$, then g preserves each of the foliations $\mathcal{W}^*$, $* = c, s, u, cs, cu$.

Following [DWX19], denote by $\mathcal{Z}^c(f)$ the subgroup of $\mathcal{Z}(f)$ consisting of elements which fix each leaf of the center foliation of f.

Let $\mathrm{MCG}(M) = \pi_0\left(\mathrm{Diff}(M)\right)$ be the mapping class group of M. Denote by $\mathcal{Z}_0(f)$ the kernel of the homomorphism $\mathcal{Z}(f) \to \mathrm{MCG}(M)$. Note that $\mathcal{Z}^c(f)$ is a subgroup of $\mathcal{Z}_0(f)$. Indeed, on the universal cover the leaf space is $\mathbb{R}^2$ and each center leaf is a line and, hence, $g \in \mathcal{Z}^c(f)$ can be homotoped to the identity along the center leaves.

Lemma 1.2. *Let $f\colon M \to M$ be a discretized Anosov flow, and suppose that the corresponding Anosov flow φ^t is transitive. Then, the group $\mathcal{Z}^c(f)$ has finite index in the kernel $\mathcal{Z}_0(f)$.*

Proof. Suppose that $g \in \mathcal{Z}_0(f)$. Since f is a discretized Anosov flow, its center foliation $\mathcal{W}^c$ is the orbit foliation of a topological Anosov flow φ^t (cf. [BFFP19, Proposition G.1]). By the preceding lemma g preserves the foliation $\mathcal{W}^c$. Thus the map g is a self orbit equivalence of the transitive Anosov flow φ^t which is homotopic to the identity. Therefore Theorem 1.1 of [BG19] applies to g.

Then, either $g \in \mathcal{Z}^c(f)$ or (see case 4 of [BG19, Theorem 1.1]) φ^t is $\mathbb{R}$-covered and there exists a map $\eta\colon M \to M$ and an integer i such that $g \circ \eta^i$ fixes every leaf of $\mathcal{W}^c$.

Since g is at least C^1, if $i \neq 0$, then g defines a non-trivial C^1 action on the weak-stable leaf space of the Anosov flow φ^t, and thus, by [Bar05, Proposition 6.6], φ^t is a finite cover of the geodesic flow on a (orientable) hyperbolic surface or orbifold Σ. That is, we are in case 4b of Theorem 1.1 of [BG19], so the map η can be chosen to be the lift of the rotation by 2π along the fiber of $T^1\Sigma$, call it r. In particular, we have $r^k = \mathrm{Id}$.

Hence, we obtained a homomorphism $\mathcal{Z}_0(f)/\mathcal{Z}^c(f) \ni [g] \mapsto i \in \mathbb{Z}/k\mathbb{Z}$ which is injective. Thus $\mathcal{Z}_0(f)/\mathcal{Z}^c(f)$ is finite. □

Lemma 1.3. *Let $f\colon M \to M$ be a discretized Anosov flow. Then for any $g \in \mathcal{Z}(f)$ and any closed center leaf $\mathcal{W}^c(x)$, there exists $k \geq 1$ such that*

$$g^k\left(\mathcal{W}^c(x)\right) = \mathcal{W}^c(x).$$

Proof. This is essentially the same proof as Lemma 23 in [DWX19], but we rewrite it since we state it in a different setting.

Let $\varphi^t\colon M \to M$ be the topological Anosov flow and $h\colon M \to \mathbb{R}^+$ be the continuous function such that $f(x) = \varphi^{h(x)}(x)$. We fix a metric on M such that the orbits of φ^t have unit speed.

Let $g \in \mathcal{Z}(f)$. Let $\tilde{\varphi}^t$, $\widetilde{f}$ and $\widetilde{g}$ be lifts of φ^t, f and g to the universal cover $\widetilde{M}$. We choose $\tilde{\varphi}^t$ and $\widetilde{f}$ to be lifts which fix each leaf of the lifted center foliation $\widetilde{\mathcal{W}}^c$ (= the flow foliation of $\tilde{\varphi}^t$). If g reverses the orientation of the orbits of φ^t, then we replace g by g^2. Thus we can assume that $\widetilde{g}$ preserves the ordering of points on any orbit of $\tilde{\varphi}^t$.

Recall that all orbits of $\tilde{\varphi}^t$ are lines. Hence a closed center leaf $\mathcal{W}^c(x)$ lifts to an orbit segment $[x, \widetilde{\varphi}^T(x)]$, $T > 0$ (where we write x for both the point $x \in M$ and a lift of it to the universal cover $\widetilde{M}$). The orbit of x under $\widetilde{f}$ is an increasing sequence of points. Hence, there exists a unique $N \geq 0$ such that $\widetilde{\varphi}^T(x)$ belongs to the orbit segment $(\widetilde{f}^N x, \widetilde{f}^{N+1}x]$. Then, for any $m \geq 1$, the points $\widetilde{g}^m\widetilde{\varphi}^T x$ belongs to the orbit segment $(\widetilde{g}^m(\widetilde{f}^N x), \widetilde{g}^m(\widetilde{f}^{N+1}x)] = (\widetilde{f}^N(\widetilde{g}^m x), \widetilde{f}^{N+1}(\widetilde{g}^m x)]$.

The center leaf $\mathcal{W}^c(g^m x)$ lifts to the orbit segment $[\widetilde{g}^m x, \widetilde{g}^m(\widetilde{\varphi}^T x)]$. By the above discussion we have $[\widetilde{g}^m x, \widetilde{g}^m(\widetilde{\varphi}^T x)] \subset [\widetilde{g}^m x, \widetilde{f}^{N+1}(\widetilde{g}^m x)]$. Hence the length of $\mathcal{W}^c(g^m x)$ is bounded by $C = (N+1)\max(h)$. Note that this bound is uniform in m.

Since there are only finitely many closed center leaves of length less than C, it follows that every closed center leaf is g-periodic.

□

Lemma 1.4. *Let $f\colon M \to M$ be a discretized Anosov flow, and suppose that the Anosov flow φ^t is transitive. Then $\mathcal{Z}(f)/\mathcal{Z}^c(f)$ is finite.*

Proof. By Lemma 1.2, since $\mathcal{Z}^c(f)$ has finite index in $\mathcal{Z}_0(f)$, it is sufficient to show that $\mathcal{Z}(f)/\mathcal{Z}_0(f)$ is finite, which we now proceed to do.

Let $g \in \mathcal{Z}(f)$. By Lemma 1.3, every closed center leaf in $\mathcal{W}^c$ is periodic under g. Now recall that each closed center leaf is a periodic orbit of the transitive Anosov flow φ^t. By [Ada87], the (conjugacy classes of) closed orbits of φ^t generate the fundamental group of M. Thus we can choose a generating set of closed orbits and choose n large enough so that g^n fixes each closed center leaf in the generating set of conjugacy classes of $\pi_1(M)$.

This implies that the element $[g^n_*] \in \mathrm{Out}(\pi_1(M))$ is the identity of the outer automorphism group of $\pi_1(M)$.

Thus g^n, seen as an element of $\mathrm{MCG}(M)$, is in the kernel of the homomorphism $\mathrm{MCG}(M) \to \mathrm{Out}(\pi_1(M))$.

A standard obstruction theory argument shows that, when M is aspherical (which is the case here, because M is 3-dimensional and supports an Anosov flow), the map $\mathrm{MCG}(M) \to \mathrm{Out}(\pi_1(M))$ is injective. Thus g^n is the identity in $\mathrm{MCG}(M)$. Hence, we conclude that $\mathcal{Z}(f)/\mathcal{Z}_0(f)$ is a torsion subgroup of $\mathrm{MCG}(M)$.

Now, since M is an irreducible 3-manifold, $\mathrm{MCG}(M)$ is virtually torsion free (see section 5 of [HM13])[2]. Thus, $\mathcal{Z}(f)/\mathcal{Z}_0(f)$ must be finite, since it is a torsion subgroup of $\mathrm{MCG}(M)$. □

We now have all needed lemmas and we can copy verbatim the proof of Theorem 5 of [DWX19] and obtain the following result that will allow us to deduce Theorem A.

Theorem 1.5. *Let f be a discretized Anosov flow on a 3-manifold M such that $\pi_1(M)$ is not virtually solvable. Suppose that f preserves a volume* Vol *on M. Then either* Vol *has Lebesgue disintegration along $\mathcal{W}^c$ or f has virtually trivial centralizer in* $\mathrm{Diff}(M)$.

Proof. As $\pi_1(M)$ is not virtually solvable, by [FP18, Theorem C], f is accessible. Because f is volume preserving, it is, thus, transitive ([Bri75]). Hence there exists a center leaf which is dense in M, which implies that the Anosov flow φ^t is also transitive. So all of the lemmas we proved above apply.

We have that $\mathcal{Z}(f)$ is virtually $\mathcal{Z}^c(f)$. Moreover, f is ergodic (because it is accessible, so it is ergodic by [HHU08, BW10]) and all the elements of $\mathcal{Z}(f)$ are volume preserving (see [DWX19, Lemma 11]).

[2] Note that McCullough [McC91] proved that $\mathrm{MCG}(M)$ is virtually torsion free for Haken manifolds and it follows from Mostow Rigidity Theorem for hyperbolic manifolds, which are the only two cases we need, since, as M supports an Anosov flow, it is either Haken or hyperbolic.

From the the proof of Theorem H of [AVW19] (see section 10.3 of [AVW19]) we have the following lemma.

Lemma 1.6. *If* Vol *has singular disintegration along the leaves of $\mathcal{W}^c$, then there exists $k \geq 1$ and a full measure set $S \subset M$ that intersects every center leaf in exactly k orbits of f.*

This lemma replaces Lemma 51 of [DWX19], and one can now copy verbatim the proof of Theorem 5 in [DWX19] (replacing T^1X with M) to obtain Theorem 1.5. □

Proof of Theorem A. If Vol has singular disintegration along the leaves of $\mathcal{W}^c$, then the conclusion of Theorem A follows from Theorem 1.5.

Otherwise, by Theorem H of [AVW19], $\mathcal{W}^c$ is absolutely continuous and $f = \psi^1$, where $\psi^t : M \to M$ is a smooth volume preserving Anosov flow. In particular, $\{\psi^t \mid t \in \mathbb{R}\} \subset \mathcal{Z}(f)$.

Now, if $g \in \mathcal{Z}^c(f)$, then, by ergodicity of f, the map g preserves Vol, and, hence, it preserves the disintegration of Vol along $\mathcal{W}^c$. Thus $g = \psi^t$ for some $t \in \mathbb{R}$.

So $\{\psi^t \mid t \in \mathbb{R}\} = \mathcal{Z}^c(f)$ and Theorem A follows from Lemma 1.4. □

2. Proofs of Theorems B and C

The two main results of [BFFP19] state that, if $f : M \to M$ is a partially hyperbolic diffeomorphism such that, either f is homotopic to the identity and M is Seifert, or that f is dynamically coherent and M is hyperbolic, then there exists $k \geq 1$ such that f^k is a discretized Anosov flow.

Since $\mathcal{Z}(f) \subset \mathcal{Z}(f^k)$, we immediately deduce from Theorem A that, under the assumptions of Theorem B or Theorem C, either $\mathcal{Z}(f)$ is virtually $\{f^n \mid n \in \mathbb{Z}\}$ or $\mathcal{Z}(f^k)$ is virtually $\mathbb{R}$ and f^k embeds into an Anosov flow for some $k \geq 1$.

Thus, in order to finish proving Theorems B and C, we only need to show that if f^k is the time-1 map of an Anosov flow which is transitive on a Seifert or hyperbolic manifold, then the centralizer of f is virtually $\mathbb{R}$.

This last step is given by the next lemma, which is in fact more general.

Lemma 2.1. *Suppose that f^k is the time-1 map of a transitive Anosov flow that is not a constant roof suspension of an Anosov diffeomorphism. Then $\mathcal{Z}(f)$ is virtually $\mathbb{R}$.*

In order to prove Lemma 2.1, we first need a result about topologically weak-mixing Anosov flows.

Lemma 2.2. *Let $\varphi^t : M \to M$ be a topologically weak-mixing Anosov flow, then, for every $n > 0$, the set of periodic orbits of φ^t that have period* not *a multiple of $1/n$ is dense in M*

Proof. This is a simple consequence of the spatial equidistribution of orbits of periods between T and $T + \varepsilon$ for weak-mixing Anosov flow.

We let $\mathcal{P}$ be the set of periodic orbits of φ^t. For any $\gamma \in \mathcal{P}$, we let $\ell(\gamma)$ be the minimal period of γ. For any map $K : M \to \mathbb{R}$ that is continuous along the orbits of φ^t, and any $\varepsilon > 0$, we have, by [PP90, Proposition 7.3],

$$\frac{\sum_{T<\ell(\gamma)\leq T+\varepsilon} \int_\gamma K}{\sum_{T<\ell(\gamma)\leq T+\varepsilon} \ell(\gamma)} \to \int_M K d\mu_{BM}, \quad \text{as} \quad T \to +\infty,$$

where μ_{BM} is the measure of maximal entropy of φ^t.

Let $n > 0$ be fixed and let $\mathcal{P}_{\notin \frac{1}{n}\mathbb{Z}}$ be the set of periodic orbits of period not a multiple of $1/n$. If $\overline{\mathcal{P}_{\notin \frac{1}{n}\mathbb{Z}}} \neq M$ then there would exists an open set U that is missed by the orbits in $\mathcal{P}_{\notin \frac{1}{n}\mathbb{Z}}$. Taking K to be a smooth approximation of the characteristic function of U and $\varepsilon < 1/n$, we would get that the left hand side of the above equation is zero along a subsequence, while the right hand side is strictly positive, as the measure of maximal entropy has full support. A contradiction. □

Proof of Lemma 2.1. Let $\varphi^t : M \to M$ be the Anosov flow such that $f^k = \varphi^1$, we will show that f itself commutes with φ^t for any $t \in \mathbb{R}$ which will prove the claim (since $\mathcal{Z}(f) \subset \mathcal{Z}(f^k)$ and $\mathcal{Z}(f^k)$ is virtually $\{\varphi^t \mid t \in \mathbb{R}\}$).

Since $f^k = \varphi^1$, we have that, for any $m \in \mathbb{Z}$ and any $x \in M$,

$$f(\varphi^m(x)) = \varphi^m(f(x)).$$

Now consider a periodic orbit γ of φ^t.

If the period of γ is irrational, then, by continuity, we have that for any $x \in \gamma$ and any $t \in \mathbb{R}$

$$f(\varphi^t(x)) = \varphi^t(f(x)).$$

On the other hand if the period of γ is rational, say p/n, $\gcd(p, n) = 1$, then for any $x \in \gamma$ and any $m \in \mathbb{Z}$, we have

$$f(\varphi^{m/n}(x)) = \varphi^{m/n}(f(x)).$$

Now, let $x \in M$, by Lemma 2.2 (which applies here because every Anosov flow which is not a suspension of an Anosov diffeomorphism by a constant roof function is topologically weak-mixing according to [Pla72]), for every $n > 1$, x can be approximated by points $y_i^n \to x$, $i \to \infty$, on periodic orbits such that the periods of y_i^n are either irrational or rational numbers p_i/q_i, $\gcd(p_i, q_i) = 1$, with $q_i \to \infty$, $i \to \infty$. As we have seen above, at each y_i^n, the map f commutes with at least φ^{m/q_i} for all $m \in \mathbb{Z}$.

Passing to the limit as $n \to +\infty$, we obtain that f commutes with every φ^t at x. Thus $\{\varphi^t \mid t \in \mathbb{R}\} \subset \mathcal{Z}(f)$, which proves the lemma. □

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