Math 35300: Sections 161 and 162. Linear algebra II **Spring 2013**

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Handout 1

Recall from lecture the following definition of a "real vector space", which we will sometimes refer to as a "vector space over $\mathbb{R}$".

Definition 1. A triple $(V, +, \cdot)$ consisting of a set V and two maps

$$V \times V \xrightarrow{+} V, \qquad (x, y) \longmapsto x + y \qquad \text{"addition"}$$

$$\mathbb{R} \times V \xrightarrow{\cdot} V, \qquad (a, x) \longmapsto ax \qquad \text{"scalar multiplication"}$$

is called a *real vector space* (or *vector space over* $\mathbb{R}$) if the following eight axioms hold for the maps $+$ and $\cdot$:

(1) $(x + y) + z = x + (y + z)$ for all $x, y, z \in V$.
(2) $x + y = y + x$ for all $x, y \in V$.
(3) There exists an element $0 \in V$ with $x + 0 = x$ for all $x \in V$.
(4) For each $x \in V$ there exists an element $-x \in V$ with $x + (-x) = 0$.
(5) $a(bx) = (ab)x$ for all $a, b \in \mathbb{R}$, $x \in V$.
(6) $1x = x$ for all $x \in V$.
(7) $a(x + y) = ax + ay$ for all $a \in \mathbb{R}$, $x, y \in V$.
(8) $(a + b)x = ax + bx$ for all $a, b \in \mathbb{R}$, $x \in V$.