Recall from lecture the following definition of a “real vector space”, which we will sometimes refer to as a “vector space over \( \mathbb{R} \).

**Definition 1.** A triple \((V, +, \cdot)\) consisting of a set \( V \) and two maps
\[
\begin{align*}
V \times V & \longrightarrow V, \quad (x, y) \longmapsto x + y \quad \text{“addition”} \\
\mathbb{R} \times V & \longrightarrow V, \quad (a, x) \longmapsto ax \quad \text{“scalar multiplication”}
\end{align*}
\]
is called a **real vector space** (or **vector space over** \( \mathbb{R} \)) if the following eight axioms hold for the maps \( + \) and \( \cdot \):

1. \((x + y) + z = x + (y + z)\) for all \( x, y, z \in V \).
2. \(x + y = y + x\) for all \( x, y \in V \).
3. There exists an element \(0 \in V\) with \(x + 0 = x\) for all \( x \in V \).
4. For each \( x \in V \) there exists an element \(-x \in V\) with \(x + (-x) = 0\).
5. \(a(bx) = (ab)x\) for all \(a, b \in \mathbb{R}, x \in V\).
6. \(1x = x\) for all \( x \in V \).
7. \(a(x + y) = ax + ay\) for all \(a \in \mathbb{R}, x, y \in V\).
8. \((a + b)x = ax + bx\) for all \(a, b \in \mathbb{R}, x \in V\).