

Warm-up Questions 1

The following warm-up questions are intended to test your understanding of some of the basic definitions and constructions introduced in lecture. Your first step to answering these should be to go back to the lecture notes and read again the appropriate definition or construction. They will not be collected or graded.

Question 1. Let A, B be sets. If for each $a \in A$ we have $a \in B$, then one writes which of the following?

- (a) $A \subset B$ (b) $A = B$ (c) $A \cup B$

Question 2. For each set A , which of the following sets is empty?

- (a) $A \cup A$ (b) $A \cap A$ (c) $A \setminus A$

Question 3. Let A, B be sets and let $a \in A$. As in lecture, we can draw a picture (or cartoon) of $A \times B$ by a rectangle (here, we are thinking of A and B as intervals, even though in general this will not be true). How would one draw a corresponding picture of the subset $\{a\} \times B$? (Please draw a picture).

Question 4. Which of the following statements is false? The map

$$\text{Id}_A: A \longrightarrow A, \quad a \longmapsto a,$$

is always

- (a) surjective (b) bijective (c) constant

Question 5. Let A, B be sets and $A \times B$ the Cartesian product. By projection onto the second factor, one understands the map π_2 as which of the following?

- (a) $A \times B \longrightarrow A$ (b) $A \times B \longrightarrow B$ (c) $B \longrightarrow A \times B$
 $(a, b) \longmapsto a$ $(a, b) \longmapsto b$ $b \longmapsto (a, b)$

Question 6. Let $f: X \longrightarrow Y$ be a map. Which of the following statements implies that f is surjective?

- (a) $f^{-1}(Y) = X$ (b) $f(X) = Y$ (c) $f^{-1}(X) = Y$

Question 7. Let $X \xrightarrow{f} Y \xrightarrow{g} Z$ be maps. Then the map $gf: X \longrightarrow Z$ is defined by which of the following?

- (a) $x \longmapsto g(f(x))$ (b) $x \longmapsto f(g(x))$ (c) $x \longmapsto g(x)(f)$

Question 8. Consider any commutative diagram of sets of the form

$$\begin{array}{ccc} & & Y \\ & \nearrow f & \uparrow h \\ X & \xrightarrow{g} & Z \end{array}$$

Then we have which of the following?

- (a) $h = gf$ (b) $f = hg$ (c) $g = fh$

Question 9. The map $f: \mathbb{R} \setminus \{0\} \rightarrow \mathbb{R} \setminus \{0\}$, $x \mapsto \frac{1}{x}$ is bijective. The inverse map $f^{-1}: \mathbb{R} \setminus \{0\} \rightarrow \mathbb{R} \setminus \{0\}$ is defined by which of the following?

- (a) $x \mapsto \frac{1}{x}$ (b) $x \mapsto x$ (c) $x \mapsto -\frac{1}{x}$

Question 10. Consider the map $\mathbb{R} \rightarrow \mathbb{R}$, $x \mapsto x^2$. Which of the following is true?

- (a) The map is surjective but not injective.
(b) The map is injective but not surjective.
(c) The map is neither surjective nor injective.