Math 35300: Sections 161 and 162. Linear algebra II **Spring 2013**
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Warm-up Questions 6

The following warm-up questions are intended to test your understanding of some of the basic definitions and constructions introduced in lecture. Your first step to answering these should be to go back to the lecture notes and read again the appropriate definition or construction. They will not be collected or graded.

Question 1. Which of the following operations cannot be made as a sequence of row and column operations?

(a) $\begin{bmatrix} 2 & 7 \\ 1 & 1 \end{bmatrix} \longmapsto \begin{bmatrix} 2 & 7 \\ 3 & 8 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 1 \\ 2 & 7 \end{bmatrix} \longmapsto \begin{bmatrix} 1 & 4 \\ 2 & 8 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 2 \\ 7 & 1 \end{bmatrix} \longmapsto \begin{bmatrix} -11 & 2 \\ 1 & 1 \end{bmatrix}$

Question 2. The rank of the real matrix

$$\begin{bmatrix} 5 & 5 & 5 \\ 5 & 5 & 5 \\ 5 & 5 & 5 \end{bmatrix}$$

is

(a) 1 (b) 3 (c) 5

Question 3. A system of linear equations with coefficients in $\mathbb{F}$ is a system of equations of the following kind:

(a) $$\begin{matrix} a_{11}x_1 + \cdots + a_{1n}x_1 = b_1 \\ \vdots \qquad \vdots \qquad \vdots \\ a_{n1}x_n + \cdots + a_{nn}x_n = b_n \end{matrix} \qquad \text{with } a_{ij} \in \mathbb{F},\ b_i \in \mathbb{F}$$

(b) $$\begin{matrix} a_{11}x_{11} + \cdots + a_{1n}x_{1n} = b_1 \\ \vdots \qquad \vdots \qquad \vdots \\ a_{n1}x_{n1} + \cdots + a_{nn}x_{nn} = b_n \end{matrix} \qquad \text{with } a_{ij} \in \mathbb{F},\ b_i \in \mathbb{F}$$

(c) $$\begin{matrix} a_{11}x_1 + \cdots + a_{1n}x_n = b_1 \\ \vdots \qquad \vdots \qquad \vdots \\ a_{n1}x_1 + \cdots + a_{nn}x_n = b_n \end{matrix} \qquad \text{with } a_{ij} \in \mathbb{F},\ b_i \in \mathbb{F}$$

Question 4. If one abbreviates a system of linear equations as $Ax = b$, then
(a) $A \in \mathsf{M}(m \times n, \mathbb{F})$, $b \in \mathbb{F}^n$.
(b) $A \in \mathsf{M}(m \times n, \mathbb{F})$, $b \in \mathbb{F}^m$.
(c) $A \in \mathsf{M}(m \times n, \mathbb{F})$, $b \in \mathbb{F}^n$ or $b \in \mathbb{F}^m$ (not fixed).

Question 5. A system of linear equations $Ax = b$ is called solvable if
(a) $Ax = b$ for all $x \in \mathbb{F}^n$.
(b) $Ax = b$ for precisely one $x \in \mathbb{F}^n$.

(c) $Ax = b$ for at least one $x \in \mathbb{F}^n$.

Question 6. If b is one of the columns of A, then $Ax = b$ is

(a) solvable in all cases
(b) unsolvable in all cases
(c) sometimes solvable, sometimes unsolvable, depending on A and b

Question 7. Let $Ax = b$ be a system of equations with square matrix A (n equations in n unknowns). Then $Ax = b$ is

(a) uniquely solvable
(b) solvable or unsolvable, depending on A, b
(c) solvable, but perhaps not uniquely, depending on A, b

Question 8. Suppose once more that $A \in \mathsf{M}(n \times n, \mathbb{F})$, that is, A is square. Which of the following conditions is (or are) equivalent to the unique solvability of $Ax = b$:

(a) $\dim \operatorname{Ker} A = 0$
(b) $\dim \operatorname{Ker} A = n$
(c) $\operatorname{rk} A = n$

Question 9. Let $A \in \mathsf{M}(n \times n, \mathbb{F})$ and $\operatorname{Ker} A \neq 0$. Then $Ax = b$ is

(a) solvable only for $b = 0$
(b) solvable for all b, possibly nonuniquely
(c) solvable only for some b, and then never uniquely

Question 10. Let A be an $n \times n$ matrix and let $Ax = b$ have two *linearly independent* solutions. Then

(a) $\operatorname{rk} A \leq n$, and the case $\operatorname{rk} A = n$ can occur for some (A, b).
(b) $\operatorname{rk} A \leq n - 1$, and the case $\operatorname{rk} A = n - 1$ can occur for some (A, b).
(c) $\operatorname{rk} A \leq n - 2$, and the case $\operatorname{rk} A = n - 2$ can occur for some (A, b).

(Hint: use the dimension formula for linear maps).