Math 35300: Sections 161 and 162. Linear algebra II **Spring 2013**
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Warm-up Questions 8

The following warm-up questions are intended to test your understanding of some of the basic definitions and constructions introduced in lecture. Your first step to answering these should be to go back to the lecture notes and read again the appropriate definition or construction. They will not be collected or graded.

Question 1. An inner product on a real vector space V is a map

(a) $\langle\ ,\ \rangle\colon\ V \times V \longrightarrow \mathbb{R}$
(b) $\langle\ ,\ \rangle\colon\ V \times V \longrightarrow V$
(c) $\langle\ ,\ \rangle\colon\ \mathbb{R} \times V \longrightarrow V$

Question 2. Positive definiteness of the inner product means that

(a) $\langle x, y\rangle > 0 \Longrightarrow x = y$.
(b) $\langle x, x\rangle > 0 \Longrightarrow x \neq 0$.
(c) $\langle x, x\rangle > 0$ for all $x \in V$, $x \neq 0$.

Question 3. Which of the following statements is (or are) correct?

(a) If $\langle\ ,\ \rangle\colon\ \mathbb{R}^n \times \mathbb{R}^n \longrightarrow \mathbb{R}$ is an inner product on the real vector space $\mathbb{R}^n$, then $\langle x, y\rangle = x_1y_1 + \cdots + x_ny_n$ for all $x, y \in \mathbb{R}^n$.
(b) If $\langle\ ,\ \rangle\colon\ \mathbb{R}^n \times \mathbb{R}^n \longrightarrow \mathbb{R}$ is an inner product on the real vector space $\mathbb{R}^n$, then $\langle x, y\rangle = (x_1y_1, \ldots, x_ny_n)$ for all $x, y \in \mathbb{R}^n$.
(c) If one defines $\langle x, y\rangle = x_1y_1 + \cdots + x_ny_n$ for all $x, y \in \mathbb{R}^n$, then one obtains an inner product on $\mathbb{R}^n$.

Question 4. By the orthogonal complement $U^\perp$ of a subspace U of the Euclidean vector space V, one understands

(a) $U^\perp := \{u \in U \mid u \perp U\}$.
(b) $U^\perp := \{x \in V \mid x \perp U\}$.
(c) $U^\perp := \{x \in V \mid x \perp U \text{ and } \|x\| = 1\}$.

Question 5. Let $V = \mathbb{R}^2$ with the standard inner product. Which of the following tuples of elements of V forms an orthonormal basis?

(a) $((1, -1), (-1, -1))$
(b) $((-1, 0), (0, -1))$
(c) $((1, 0), (0, 1), (1, 1))$

Question 6. Which of the following conditions on a linear map $f\colon\ V \longrightarrow W$ of one Euclidean space into another is equivalent to f being orthogonal?

(a) $\langle f(x), f(y)\rangle > 0$ for all $x, y \in V$.
(b) $\langle x, y\rangle = 0 \Longleftrightarrow \langle f(x), f(y)\rangle = 0$.
(c) $\|f(x)\| = \|x\|$ for all $x \in V$.

Question 7. For which subspaces $U \subset V$ is the orthogonal projection $P_U\colon\ V \longrightarrow U$ an orthogonal map?

(a) for each U (b) only for $U = V$ (c) only for $V = \{0\}$

Question 8. Which of the following matrices is (or are) orthogonal?

(a) $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

Question 9. Which of the following arguments correctly explains why $(\mathbb{N}, +)$ fails to be a group?

(a) For natural numbers we have $n + m = m + n$, but this is not one of the group axioms, so $(\mathbb{N}, +)$ fails to be a group.
(b) The operation $\mathbb{N} \times \mathbb{N} \longrightarrow \mathbb{N}$, $(n, m) \longmapsto n + m$, is not defined for all integers, because the negative numbers do not belong to $\mathbb{N}$. Therefore, $(\mathbb{N}, +)$ fails to be a group.
(c) The third group axiom (existence of inverses) is not satisfied, since, for example, for $1 \in \mathbb{N}$ there exists no $n \in \mathbb{N}$ with $1 + n = 0$. Therefore, $(\mathbb{N}, +)$ fails to be a group.

Question 10. For $k > 0$ we have

(a) $SO(2k) \subset O(k)$.
(b) $SO(2k) \subset O(2k)$, but $SO(2k) \neq O(2k)$.
(c) $SO(2k) = O(2k)$, because $(-1)^{2k} = 1$.