Math 35300: Sections 161 and 162. Linear algebra II **Spring 2013**

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Warm-up Questions 2

The following warm-up questions are intended to test your understanding of some of the basic definitions and constructions introduced in lecture. Your first step to answering these should be to go back to the lecture notes and read again the appropriate definition or construction. They will not be collected or graded.

Question 1. Let $n \geq 1$. Then $\mathbb{R}^n$ consists of

(a) n real numbers
(b) n-tuples of real numbers
(c) n-tuples of vectors

Question 2. Which of the following statements is not an axiom for real vector spaces?

(a) For all $x, y \in V$ we have $x + y = y + x$.
(b) For all $x, y, z \in V$ we have $(x + y) + z = x + (y + z)$.
(c) For all $x, y, z \in V$ we have $(xy)z = x(yz)$.

Question 3. For the multiplication of complex numbers we have $(x+yi)(a+bi) =$

(a) $xa + ybi$
(b) $xy + yb + (xb - ya)i$
(c) $xa - yb + (xb + ya)i$

Question 4. In a vector space V over a field $\mathbb{F}$ scalar multiplication is given by a map

(a) $V \times V \longrightarrow \mathbb{F}$
(b) $\mathbb{F} \times V \longrightarrow V$
(c) $\mathbb{F} \times \mathbb{F} \longrightarrow \mathbb{F}$

Question 5. Which formulation below can be completed correctly to give the definition of the concept of a real vector space?

(a) A set V is called a *real vector space* if there exist two maps $+ : \mathbb{R} \times V \longrightarrow V$ and $\cdot : \mathbb{R} \times V \longrightarrow V$ so that the following eight axioms are satisfied ...
(b) A set of real vectors is called a *real vector space* if the following eight axioms are satisfied ...
(c) A triple $(V, +, \cdot)$ in which V is a set, and $+$ and $\cdot$ are maps $V \times V \longrightarrow V$ and $\mathbb{R} \times V \longrightarrow V$, respectively, is called a *real vector space* if the following eight axioms are satisfied ...

Question 6. Which of the following statements is true? If V is a vector space over a field $\mathbb{F}$ then

(a) $\{x + y \mid x \in V, y \in V\} = V$.
(b) $\{x + y \mid x \in V, y \in V\} = V \times V$.
(c) $\{ax \mid a \in \mathbb{F}, x \in V\} = \mathbb{F} \times V$.

Question 7. Which of the following statements is true?

(a) If U is a subspace of V, then $V \setminus U$ is also a subspace of V.

(b) There exists a subspace U of V for which $V \setminus U$ is also a subspace, but $V \setminus U$ is not a subspace for all subspaces U.
(c) If U is a subspace of V, then $V \setminus U$ is never a subspace of V.

Question 8. Which of the following subsets $U \subset \mathbb{R}^n$ is a vector subspace?

(a) $U = \{x \in \mathbb{R}^n \mid x_1 = \cdots = x_n\}$
(b) $U = \{x \in \mathbb{R}^n \mid x_1^2 = x_2^2\}$
(c) $U = \{x \in \mathbb{R}^n \mid x_1 = 1\}$

Question 9. On restriction of scalar multiplication to the scalar domain $\mathbb{R} \subset \mathbb{C}$, a complex vector space $(V, +, \cdot)$ becomes a real vector space $(V, +, \cdot|\mathbb{R} \times V)$. In particular, $V := \mathbb{C}$ can itself be regarded as a real vector space in this way. Do the imaginary numbers $U = \{yi \in \mathbb{C} \mid y \in \mathbb{R}\}$ then form a vector subspace?

(a) Yes, because then $U = \mathbb{C}$.
(b) Yes, because $0 \in U$ and when $a \in \mathbb{R}$ and $xi, yi \in U$, we also have $(x+y)i \in U$ and $axi \in U$.
(c) No, because ayi does not need to be imaginary, since for example $i^2 = -1$.

Question 10. How many vector subspaces does $\mathbb{R}^2$ have?

(a) two: $\{0\}$ and $\mathbb{R}^2$
(b) four: $\{0\}$, the "axes" $\mathbb{R} \times 0$ and $0 \times \mathbb{R}$, and $\mathbb{R}^2$ itself.
(c) infinitely many